THE UNIVERSITY OF ALBERTA

AN INVESTIGATION OF HIGH SPIN STATES IN ^{55}Fe

by

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF SCIENCE

DEPARTMENT OF PHYSICS

EDMONTON, ALBERTA

FALL, 1969

UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled AN INVESTIGATION OF HIGH SPIN STATES IN ^{55}Fe submitted by Aadu Andres Pilt in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

A study of the gamma radiation following the $^{55}Mn(p,n\gamma)^{55}Fe$ reaction has yielded level energies, decay branching ratios, level spins, and multipole mixing ratios of the transitions for ^{55}Fe. Spins and multipole mixing ratios were determined by comparing experimental gamma-ray angular distributions with the predictions of the compound nuclear statistical model. The following spin assignments of the ^{55}Fe levels were made: 1.316 MeV (7/2), 1.408 MeV (7/2), 2.144 MeV (5/2), 2.211 MeV (9/2), 2.301 MeV (9/2), 2.470 MeV (3/2) and 2.578 MeV (5/2). These predictions were compared with the shell-model calculations of Vervier and Ohnuma as well as with the predictions of the particle-core excitation model.

The theory of the compound nuclear statistical model applied to gamma ray distributions and a description of the program used in the analysis is given in an appendix.

ACKNOWLEDGMENTS

I would like to thank my supervisor, Dr. William C. Olsen, for his encouragement and advice during the course of this work, and for convincing me that an amplifier is <u>not</u> just a "black box".

I am especially grateful to Dr. Douglas M. Sheppard and Dr. Peter J. Twin for their assistance with all phases of the experiment, from conception to completion, and for the many discussions (and disagreements) I had with them on all aspects of nuclear physics.

To Georges Carola, who so unselfishly gave up his new-found bliss to share the frustrations and fatigue of so much of the experimental work, who so patiently taught me (often at 5 A.M.) the finer points of van de Graaffia, and who was eternally consigned to suffer my unendurable French, I can only say: Merci, Georges, et vive la Corse libre!

To my other colleagues in Centre Port: Woon Chung, Paul Gutowski and Ed Wong, as well as the irrepressible Dr. Brian Sowerby, my sincere thanks for their innumerable acts of assistance.

May Dr. Kenneth Dawson's "pets" find true happiness in life together, and may they continue to serve the entire Centre as obediently and graciously as they served me.

The technical staff, Jock Elliott, Lars Holm, Paul Karvonen, Ron Popik, and Harold Fodchuk, I thank and congratulate; for their respective <u>curae</u> managed to survive intact just long enough for me to glean all my data.

And a final thanks to Mrs. Kay Baert for her efficient and elegant work in typing the manuscript, and Miss Elsie Hawirko for coming to the rescue with some last-minute changes.

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CHAPTER 1

BACKGROUND

1.1 Theoretical Motivation

The nuclei with N=29 have been recognized by several investigators (Ra 63, Oh 66, Ve 66) as being particularly useful for a systematic study of the 1f-2p shell nuclei because of their simple $\nu 2p_{3/2}$ configuration and accessibility via nuclear reactions on stable neighbouring isotopes. The level schemes of many of these nuclei have been determined from the (d,p) reactions (Fu 63, Ma 64) but comparatively little work has been done on the gamma ray decay and spin assignments of their excited states. The latter are useful to determine since their comparison with predictions from various nuclear models (meaning theoretical calculations) forms a sensitive test of the validity or range of applicability of the model.

There are two main approaches to a theoretical study of these interesting nuclei. Ohnuma (Oh 66) and Vervier (Ve 66) have used the shell model with configuration mixing using a spin-dependent effective p-n inreraction

$$V_{pn}=-V_0'\{(1-\alpha')+\alpha(\sigma_p.\sigma_n')\}f(r)$$

with Ohnuma using a Gaussian form for the radial function f(r) and Vervier a δ-function. The parameters V_0 and α were chosen to fit the observed level spacings of the neighbouring even N=28 nuclei. The two investigations differ again in the size of the matrices considered; Ohnuma considers levels of spin up to 13/2 in the odd-A nuclei whereas Vervier truncates at J=7/2. The results obtained by both are in reasonable agreement with the known data on excitation energies but too little information on electromagnetic properties and spins was known to attempt a quantitative appraisal of the merits of the respective calculations.

The second approach assumes that the low-lying states of the odd-A

nuclei are well described by coupling the $p_{3/2}$ neutron to the ground or an excited state of the neighbouring even-even nucleus (core-particle coupling). The simplest calculation assumes that the core is able to perform collective quadrupole oscillations. The nature of the excited states of the core and the coupling are then assumed known and the excited states of the odd-A nucleus may be predicted. Assuming further that the coupling is weak and that the single-particle energies are well separated leads to the "centre-of-gravity theorem" of Lawson and Uretsky (La 57) which is a test of the validity of this model (see Ch. 4). The failure of this theorem in several cases (Th 64) has indicated that the above assumptions are not very accurate; de-Shalit had earlier proposed a generalisation where the nature of the core is not made explicit and where the coupling is not necessarily weak (Sh 61); this has met with fair success in describing the copper nuclei where Z=29.

To demonstrate the relative merits of these methods, spin determinations and a study of the electromagnetic properties of the excited states of the N=29 nuclei are necessary. An investigation of the nuclei ^{51}Ti and ^{53}Cr is at present under way (Ca 69); the present report considers the spins and decay of the levels of ^{55}Fe, using the $^{55}Mn(p,n\gamma)^{55}Fe$ reaction.

1.2 Previous Experimental Work on ^{55}Fe

Experimental investigations on ^{55}Fe have mainly been part of studies of the (d,p) or (p,d) reactions on a number of medium-A even nuclei (Fu 63, Ma 64, Sp 61, Gl 66). These studies have revealed a large number of excited states below 3 MeV, some of them exhibiting charged particle angular distributions which were either isotropic or characteristic of large angular momentum transfer, both indicating the presence of high spin states. The observed values of ℓ_n together with the Lee-Schiffer rules

(Le 64) allowed a few spin assignments to be made.

Gamma ray studies have included an investigation of the de-excitation of ^{55}Fe following the $\beta+$ decay of ^{55}Co (Fi 66) which includes measured branching ratios and tentative spin assignments based on the decay scheme and previous data. Bauer and Deutsch (Ba 60) studied the $\beta+$ decay of polarized ^{55}Co at low temperatures and from the angular distribution and linear polarization of the cascade gamma rays, found the spin of the 930 keV level (5/2-). Gemmel _et al_. (Ge 66) studied the p-γ correlation in the ^{54}Fe (d,pγ) reaction for the 412 keV level and from the isotropic distribution, concluded a spin of 1/2. Coté (Co 64) reached a similar conclusion from the ^{54}Fe(n,$\gamma\gamma$) correlation. A previous ^{55}Mn(p,nγ) experiment (Iy 67) found no evidence for the levels at 510 and 690 keV suggested by Kim (Ki 63) and made a few tentative spin assignments from the gamma ray angular distributions.

1.3 The Compound Nuclear Statistical Model

The study of angular distributions of gamma rays from aligned nuclear states can frequently yield information on level spins. The CN statistical model of Sheldon and van Patter (Sh 66), described later in this section and more fully in Appendix I, can be used to analyze gamma distributions following reactions of the form (p,nγ), (α,nγ), (p,p'γ), and (n,n'γ). Such a method has proved fruitful in some recent investigations of nuclei in the A=40 to 70 mass region but has yet seen comparatively little use. (See, for example, Bi 68, Sa 67, Ab 67, Mc 69 and Tw 69.)

In the statistical model approach, the reaction is assumed to pass through a large number of compound levels in the intermediate nucleus. Some of these levels, those formed by incoming particles having large orbital angular momentum, will have quite high spin (say J1) and will be aligned, that is, have only the lower magnetic substates populated. The

subsequent emission of (say) a neutron of low energy will lead to a level in the residual nucleus of spin J1±1/2, assuming that s-waves will predominate in the outgoing channel, having essentially the same alignment as the compound state. The condition ℓ_n=0 ensures that de-alignment of the CN state is minimized. To fulfill this requirement, the neutron energy must be near zero; this can be arranged by suitably selecting the bombarding energy to make the reaction proceed near threshold.

The probability of emission of a particle of a specified partial wave ℓ and energy E is given in the statistical model by the transmission coefficient $T_\ell^J(E)$. As shown in Appendix I, the angular distribution of a gamma ray de-exciting a level in the residual nucleus is proportional to a sum over products of vector-coupling coefficients (describing the probability of formation of a level of given spin) and elements of the reaction matrix. These latter may be rewritten in terms of the transmission coefficients (TC). Dominating are terms of the form $T_{\ell=\ell'}/T_{\ell=0}$ for each outgoing partial wave ℓ'. Very close to threshold, the T_ℓ are quite small for higher partial waves and there will be negligible contributions from these in the reaction. This is essentially what keeps the residual nucleus aligned as there is no opportunity for the higher magnetic substates to become populated; this would require emission of particles of large angular momentum.

For the sake of accuracy in the analysis the higher partial waves should be considered as well, especially if the reaction is observed several hundred keV above threshold (as may be necessary if the yield is small). Then the $T_{\ell=1}$ and $T_{\ell=2}$ transmission coefficients become non-negligible; the higher ones may still be ignored as they are extremely small. Figure 1 shows plots of the ratios $T_{\ell=1}/T_{\ell=0}$ and $T_{\ell=2}/T_{\ell=0}$ as a function of the energy above threshold for the $^{55}Mn(p,n\gamma)^{55}Fe$ reaction.

As described later, several sets of transmission coefficients were

Figure 1. *Plots of the ratio of neutron transmission coefficients* $T_{\ell=1}/T_{\ell=0}$ *and* $T_{\ell=2}/T_{\ell=0}$ *as a function of outgoing energy.*

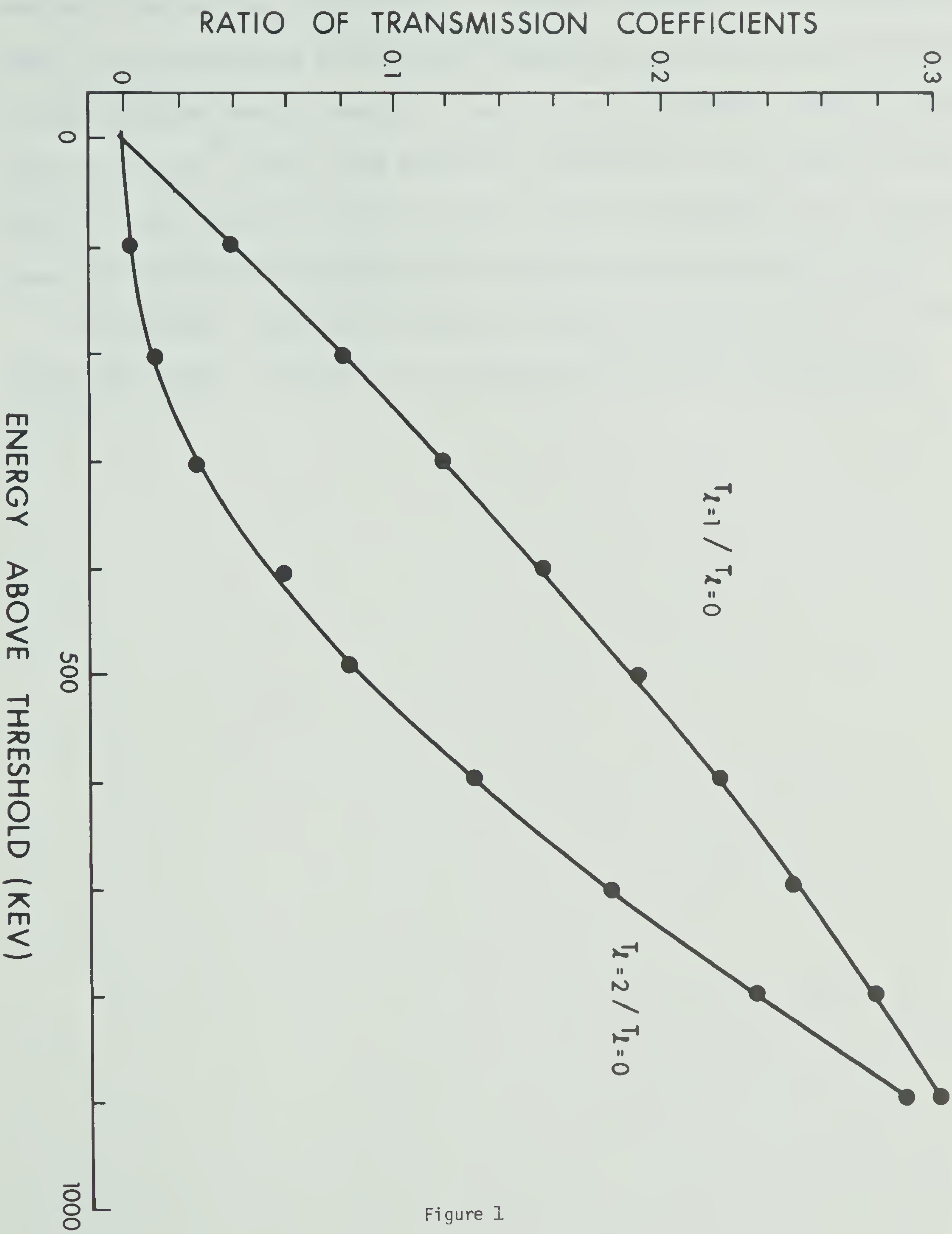

RATIO OF TRANSMISSION COEFFICIENTS
0
0.1
0.2
0.3
ENERGY ABOVE THRESHOLD (KEV)
0
500
1000
$T_{\ell=1} / T_{\ell=0}$
$T_{\ell=2} / T_{\ell=0}$

Figure 1

obtained from various calculations. Although different, the statistical model calculations were sufficiently insensitive to these values that all of them produced nearly identical results when the angular distributions were calculated. Much of the power and usefulness of this method arises from the fact that the results are only weakly dependent on the inexactly known optical model parameters which determine the transmission coefficients.

The computer code MANDY written by Sheldon (Sh 66) was used for the statistical model calculations of the gamma ray angular distributions.

CHAPTER 2

EXPERIMENTAL PROCEDURE

2.1 Apparatus

Protons from the University of Alberta 5.5 MeV van de Graaff were used to bombard thick targets of natural manganese. The beam, after terminal analysis and focussing, was bent through 90° by an analyzing magnet and defined on the target by a double-focussing quadrupole pair and a collimator of diameter 0.15 cm. The target holder was a 5 cm diameter cylindrical piece of Perspex faced at 30° to the beam, and had a 1.35 cm diameter hole to allow the beam to pass through. The holder could be directly fastened to the beam line and the target attached to the former with a 2 mm thick aluminium plate which also served as charge collector. The targets were prepared by mixing a small amount of powder into a glue formed by dissolving polyurethane in benzene, and applying the mixture to an 0.025 cm thick Ta backing.

Bombarding energies were chosen so that the reaction proceeded near threshold for the production of each level of ^{55}Fe of interest. This ensured minimal de-alignment of the CN state and anisotropic gamma ray angular distributions for high spin states.

Gamma rays were observed with a movable 8 cc GeLi detector and a fixed 7.5 cm x 7.5 cm Harshaw NaI(Tl) scintillation crystal used initially as a monitor. A 45 cc GeLi detector was also used to obtain better statistics for some of the weaker transitions. The resolution obtained with the GeLi detectors was typically 3.5 keV FWHM for the 1.332 MeV ^{60}Co full energy peak. Signals from the GeLi detector were fed into a pre-amplifier†-amplifier†† assembly and sent to an analogue-to-digital

† Canberra Model 1408 A/B
†† TENNELEC Model TC203BLR Linear Amplifier

converter†††, and thence to an SDS 920 computer equipped with display and light pen. The spectrum from the NaI crystal was sent into a separate pulse height analyzer††††, the contents of which could be dumped into the computer memory at the end of a run.

2.2 Acquisition and Analysis of Data

Following a preliminary run to determine the gamma ray yield and a decay scheme for ^{55}Fe, angular distributions of the gamma rays were taken at five angles (0, 30, 34, 60, and 90 degrees) for bombarding energies near threshold for each level of interest. Two measurements were usually taken per angle, each run requiring some three hours; some runs were later repeated with the 45 cc detector. The average beam on target was kept at ~200 nA as higher currents would have resulted in excessive dead time corrections.

Gamma rays from the decay of ^{55}Fe were determined i) by observing new transitions as the bombarding energy was increased, corresponding to the population of new levels and ii) by fitting them consistently into a decay scheme based on the known levels of ^{55}Fe. Contaminants from carbon and oxygen were no problem because of their large negative Q-value of the (p,n) reaction. The only observed gammas not from ^{55}Fe were i) from the ^{55}Mn(p,p'γ) reaction, from which the first three excited states of ^{55}Mn were found to lie at 0.136, 0.981, and 1.303 MeV, in agreement with the previously known data (Le 67); ii) a 1237 keV gamma ray from the decay of the first excited state of ^{56}Fe from the ^{55}Mn(p,γ) reaction and iii) broad gamma rays at 598 and 693 keV from the ^{72,74}Ge(n,n'γ) reactions; the latter being due to neutrons from the (p,n) reaction interacting with

†††Technical Measurements Corporation (TMC) Model 217A

††††TMC Model 210-1024

the germanium in the detector.

The strong 412 keV gamma ray from the first excited state of ^{55}Fe (J=1/2-) was used as an internal monitor because of its isotropic distribution. Initially, the NaI spectrum was used as an additional monitor (being fixed at 90°). However, the internal monitor was used for all the normalizations. Not only is this a better monitor but corrections for dead time and correlation table anisotropies are automatically accounted for. A typical GeLi spectrum taken at 4.25 MeV bombarding energy is shown in Figure 2.

The intensities of the gamma ray peaks were obtained by summing and subtracting the background directly from the computer display. As a check, some of these were compared to the results obtained from a peak fitting program written by Tepel (Te 66). The results agreed to within the statistical errors and the first method was generally used because of its convenience.

The experimental angular distributions were fitted using lease squares to an expansion $W(\theta)=a_0(1+a_2P_2(\cos\theta)+a_4P_4(\cos\theta))$ with the Legendre coefficients a_2, a_4 treated as parameters. These values were then compared to the predictions of MANDY for various spin sequences and mixing ratios. The following sets of transmission coefficients were used to obtain the MANDY predictions:

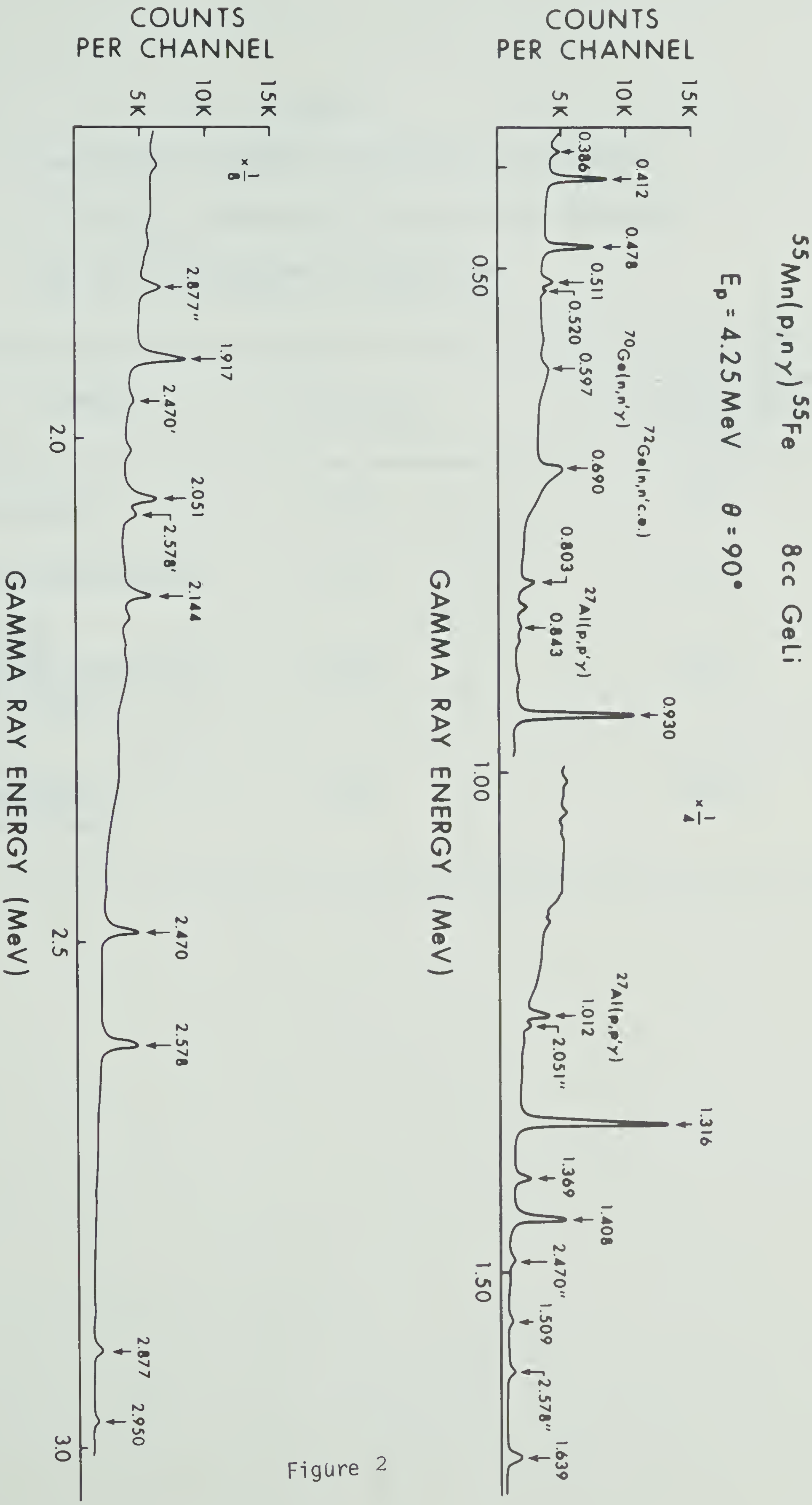
i) Auerbach-Perey coefficients from the ABACUS-II code (Au 66)

ii) Björklund-Fernbach coefficients from the same code

iii) using Rosen's (Ro 66) optical model parameters in a Hauser-Feshbach program by Davison (Da 69).

Although the absolute magnitudes of the TC differed by a factor of three, the relevant quantities $T_{\ell=1}/T_{\ell=0}$ agreed to within about 30%. The MANDY calculations moreover were extremely insensitive to these changes (see Table 1) showing the usefulness of the method.

Figure 2. *A typical gamma-ray spectrum taken at a bombarding energy of 4.25 MeV with the 8 cc GeLi detector.*



$^{55}Mn(p,n\gamma)^{55}Fe$
8cc GeLi
$E_p = 4.25$ MeV
$\theta = 90°$
COUNTS PER CHANNEL
15K
10K
5K
0.386
0.412
0.478
0.511
0.520
0.597
$^{70}Ge(n,n'\gamma)$
0.690
$^{72}Ge(n,n'c.e.)$
0.803
$^{27}Al(p,p'\gamma)$
0.843
0.930
0.50
1.00
$\times\frac{1}{4}$
$^{27}Al(p,p'\gamma)$
1.012
2.051"
1.316
1.369
1.408
2.470"
1.509
2.578"
1.639
1.50
GAMMA RAY ENERGY (MeV)
COUNTS PER CHANNEL
15K
10K
5K
$\times\frac{1}{8}$
2.877"
1.917
2.470'
2.051
2.578'
2.144
2.470
2.578
2.877
2.950
2.0
2.5
3.0
GAMMA RAY ENERGY (MeV)

Figure 2

TABLE 1

Sensitivity of MANDY *calculations to the ratio of* $T_{\ell=1}/T_{\ell=0}$ *obtained at 200 keV above threshold from three independent calculations.*

Source	$T_{\ell=1}/T_{\ell=0}$	a_2 for 7/2-3/2 transition at 200 keV above threshold
N. Davison	0.12	0.102
Auerbach-Goldfarb	0.10	0.110
ABACUS Björklund-Fernbach parameters	0.08	0.097
ABACUS Perey-Buck coefficients	0.09	0.100

In a few cases, unique spin assignments could be determined by superimposing the experimental Legendre coefficients directly onto the multipole ellipses predicted by MANDY. This however gives no indication of the goodness of fit of other possible spin assignments. Let us define χ^2 by

$$\chi^2 = \frac{1}{n-1} \sum_{i=1}^{n} \frac{|e_i - t_i|^2}{(\Delta e_i)^2}$$

with e_i and t_i the experimental and predicted data points, respectively, Δe_i the statistical uncertainty in point e_i, and n the number of degrees of freedom. Then it is customary to reject a hypothesis only if it falls outside the limits of χ^2 at the 0.1% confidence limit; implying only one chance in 1000 of rejecting a correct hypothesis. Accordingly, χ^2 was calculated for the experimental and predicated a_2 and a_4 coefficients for all possible spin sequences and mixing ratios. These χ^2 plots are given together with the angular distributions for the more interesting gamma rays in the next section.

CHAPTER 3

EXPERIMENTAL RESULTS

3.1 General

In this chapter we consider in detail the results summarized in Tables 2 - 4 consisting of the measured γ-branching ratios, Legendre coefficients of the angular distributions, and deduced spins and mixing ratios. Figure 4 gives a decay scheme of ^{55}Fe together with the level spins as deduced here and values of ℓ_n from previous work (Fu 63, Ma 64). The following diagrams show selected angular distributions with the two "best fits" and the corresponding χ^2 plots. The convention used here for the sign of the mixing ratio is that defined by Rose and Brink (Ro 67).

3.2 Determination of Level Energies

A calibration curve was constructed from a ^{22}Na - ^{60}Co spectrum and used for the determination of gamma ray energies below 2 MeV. The energies of ground state transitions so found were checked, whenever possible, with the sum of the cascades via intermediate levels. These were invariably found to agree within < 1 keV. The energies of states above 2 MeV which decay by cascade were determined from the first decay to a state of accurately known energy; those which only decay to the ground state were determined by extrapolating the calibration. These were checked by measuring the energy of the second, and where possible, first escape peaks. The errors in these level energies are ± 2 keV.

3.3 Branching Ratio Measurements

Approximate branching ratios were determined from all the observed gamma ray intensities at 60°. Since the a_4 of the distributions was for all cases very small, measuring the intensity at this angle rather than

TABLE 2

Gamma-ray branching ratios obtained in the present experiment, and comparison to results of Fischbeck et. al. (Fi 66).

Initial State	Residual State	This Expt.	Fischbeck
412	0	100	100
930	0	98±1	99
	412	2±1	1
1316	0	96±1	92
	930	4±1	8
1408	0	46±5	49
	930	54±5	43
	1316	*	8
1918	0	68±3	not observed
	412	32±3	
2051	0	23±2	not observed
	412	77±2	
2144	0	17±2	100
	930	58±4	
	1316	25±3	
2211	1316	(20)	not observed
	1408	100	
2301	930	80±10	not observed
	1316	20±10	
	1408	(10)	
2470	0	(100)	not observed
2578	0	(100)	100
2877	0	100	60
	930		40
2950	0	100	55
	1316		45

*Not observed since energy (92 keV) was below cutoff.

TABLE 3

Legendre coefficients, corrected for solid angle effects, fitted to angular distributions of gamma rays de-exciting levels of ^{55}Fe. The initial states were populated near threshold, implying alignment of high spin states as discussed in the text.

Bombarding energy (MeV)	Transition	E_γ (MeV)	a_2	a_4
2.75	0.930 → 0	0.930	0.00 ± 0.02	0.01 ± 0.02
	1.316 → 0	1.316	0.11 ± 0.02	0.02 ± 0.02
	1.316 → 0.930	0.386	0.07 ± 0.16	-0.03 ± 0.17
	1.408 → 0	1.408	0.13 ± 0.02	0.05 ± 0.03
	1.408 → 0.930	0.478	0.03 ± 0.04	-0.02 ± 0.04
3.40	1.917 → 0	1.917	-0.06 ± 0.07	-0.07 ± 0.07
	2.051 → 0	2.051	-0.09 ± 0.06	0.06 ± 0.06
	2.051 → 0.412	1.639	0.00 ± 0.03	0.04 ± 0.04
	2.140 → 0	2.140	-0.10 ± 0.06	-0.05 ± 0.06
	2.211 → 1.408	0.803	-0.54 ± 0.05	0.04 ± 0.06
3.50	2.211 → 1.408	0.803	-0.30 ± 0.05	0.00 ± 0.05
	2.301 → 0.930	1.371	0.39 ± 0.05	0.03 ± 0.06
3.85	2.470 - 0.	2.470	-0.05 ± 0.02	-0.01 ± 0.03
	2.578 - 0.	2.578	-0.05 ± 0.04	-0.02 ± 0.05

TABLE 4

Spin assignments and multipole mixing ratios determined in the present work.

Initial State	Final State	Spin sequence assumed	Mixing ratio solution
1316	0	7/2 - 3/2	E2
1408	0	7/2 - 3/2	E2
2140	0	5/2 - 3/2	$1.0^{+\infty}_{-1.3}$
2210	1408	9/2 - 7/2	$0.19^{+0.07}_{-0.09}$ or $2.75^{+0.75}_{-0.50}$
2310	930	9/2 - 5/2	E2
2470	0	3/2 - 3/2	$-1.0^{+1.0}_{-\infty}$
2579	0	5/2 - 3/2	$1.0^{+\infty}_{-1.0}$

Figure 3. *Relative photopeak efficiency of the 8 cc GeLi detector for a range of gamma ray energies from 0.5 to 3.0 MeV used in the branching ratio determinations.*

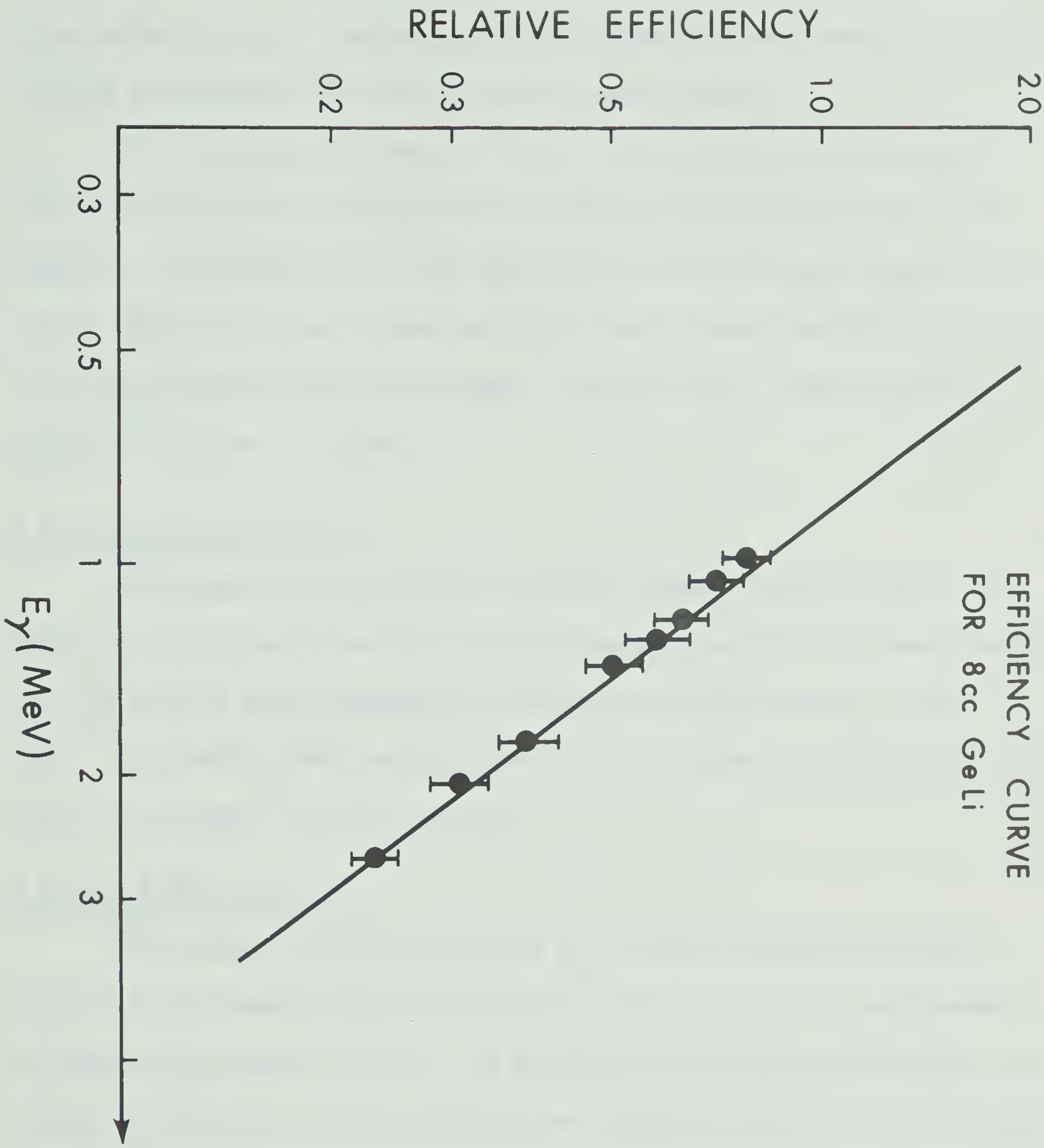

RELATIVE EFFICIENCY
2.0
1.0
0.5
0.3
0.2
EFFICIENCY CURVE
FOR 8cc GeLi
0.3
0.5
1
2
3
E_γ(MeV)

Figure 3

55° (where P_2=0) yields a maximum error for the largest a_2 measured (0.5) of less than 5%. Most of the a_2's were only ~0.1, corresponding to an error of < 1%. The uncertainties quoted for the branching ratios include both statistical errors and the above effect.

The relative efficiency of the 8 cc GeLi detector (with which the branching ratio measurements were done) was measured using a ^{56}Co source. The intensities of the gamma rays from ^{56}Co decay ranging from 734 to 3254 keV are well known and their large number permits an accurate efficiency determination to be made. The efficiency curve for the 8 cc detector is given in Figure 3.

3.4 The 0.412 MeV Level

The gamma ray distribution from this level was very nearly isotropic. Its stripping pattern in the ^{54}Fe(d,p) reaction is characteristic of ℓ_n=1 (Ma 64) and gamma-gamma correlations (Co 64) favour a spin of 1/2-. As a J=1/2 level decays by an isotropic gamma ray, this transition serves as an ideal internal monitor.

3.5 The 0.930 MeV Level

This level is characterized by ℓ_n=3 angular momentum transfer (Fu 63) and is known to have J=5/2 from the polarized ^{55}Co measurements of Bauer and Deutsch (Ba 60). The principal decay (98%) is to the ground state; its distribution was found to be isotropic, which is not unexpected as it was populated far above threshold. The 520 keV transition was too weak to allow an angular distribution to be made.

3.6 The 1.316 MeV Level

Most of the decay of this level (96%) is to the ground state, the remaining going to the 0.930 MeV level. The angular distribution of the 1316 keV transition was anisotropic (a_2=0.11±0.02) and Figure 4a shows

Figure 4 *Decay scheme and spins of levels in* ^{55}Fe *as deduced from the present experiment. The values of* ℓ_n *are taken from Fu 63 and Ma 64.*

Figure 4a *Angular distribution and fitted* χ^2 *vs. arc tan* δ *plots for the 1.316→0.0 MeV transition.*

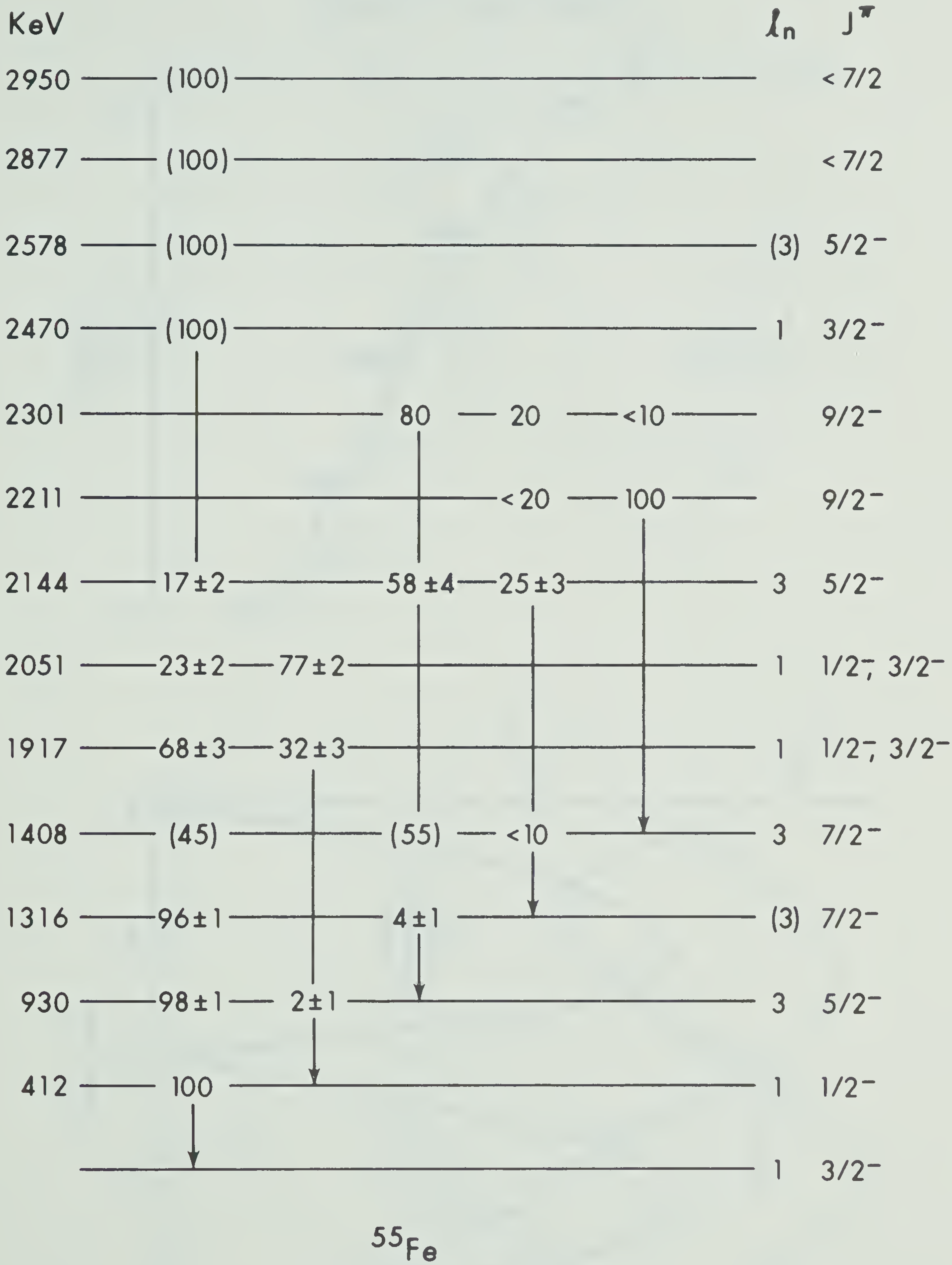
Figure 4b *Angular distribution and fitted* χ^2 *vs. arc tan* δ *plots for the 1.408→0.0 MeV transition.*

Figure 4c *Angular distribution and fitted* χ^2 *vs. arc tan* δ *plots for the 2.144→0.0 MeV transition.*

Figure 4d *Angular distribution and fitted* χ^2 *vs. arc tan* δ *plots for the 2.211→1.408 MeV transition.*

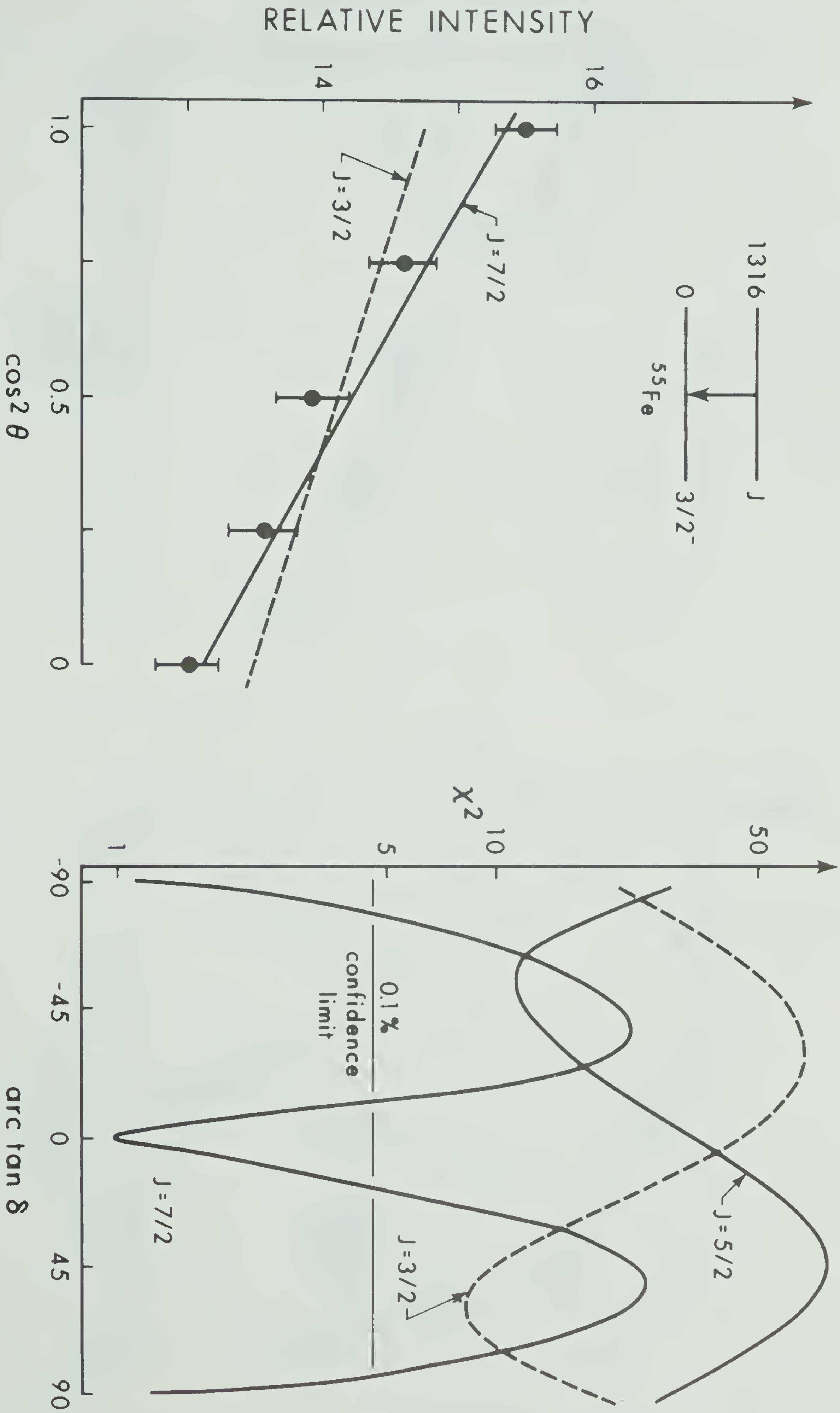
Figure 4e *Angular distribution and fitted* χ^2 *vs. arc tan* δ *plots for the 2.301→0.930 MeV transition.*

Figure 4f *Angular distribution and fitted* χ^2 *vs. arc tan* δ *plots for the 2.470→0.0 MeV transition.*



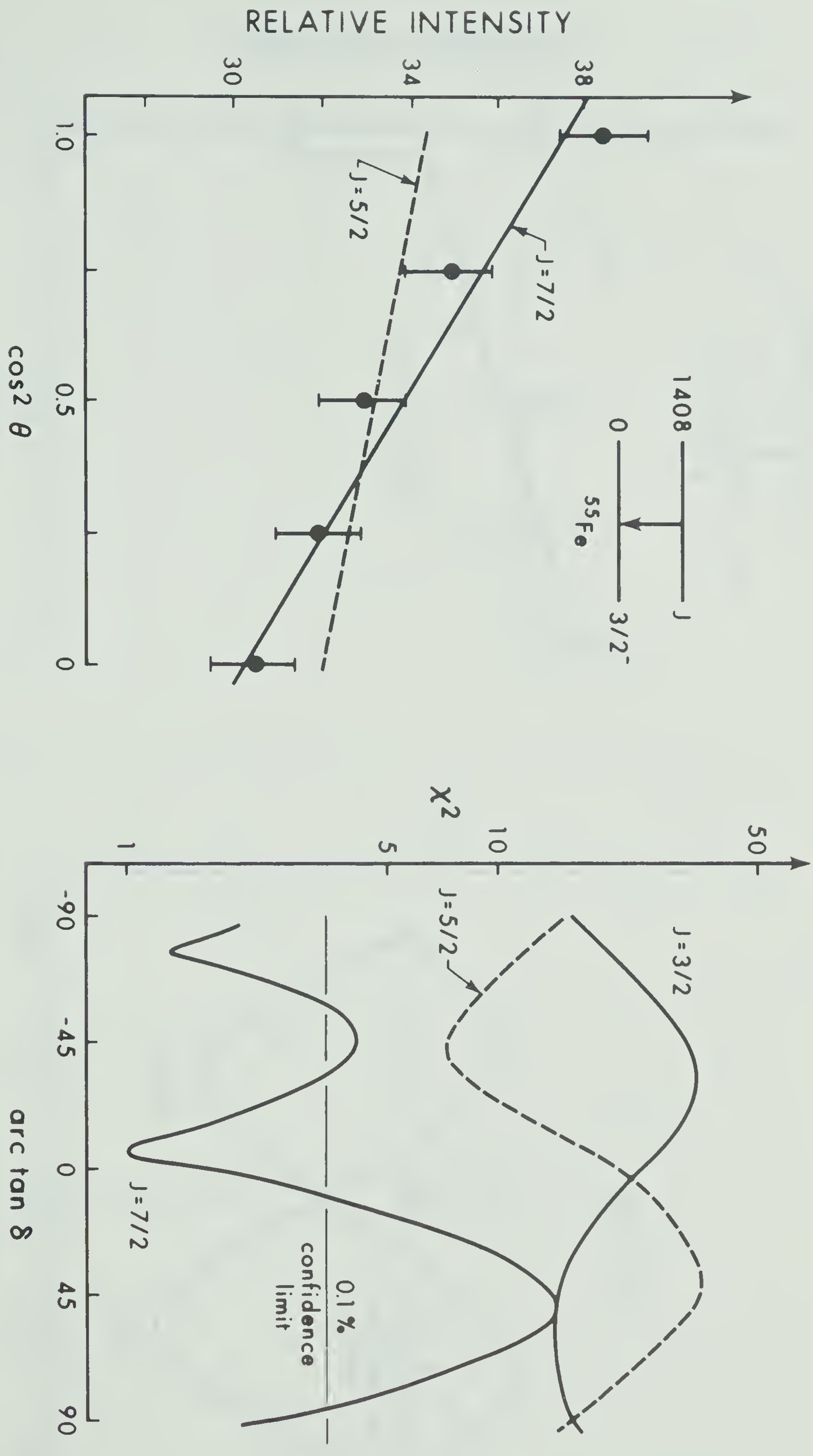
KeV
l_n
J^π
2950 (100) < 7/2
2877 (100) < 7/2
2578 (100) (3) 5/2⁻
2470 (100) 1 3/2⁻
2301 80 20 <10 9/2⁻
2211 <20 100 9/2⁻
2144 17±2 58±4 25±3 3 5/2⁻
2051 23±2 77±2 1 1/2⁻, 3/2⁻
1917 68±3 32±3 1 1/2⁻, 3/2⁻
1408 (45) (55) <10 3 7/2⁻
1316 96±1 4±1 (3) 7/2⁻
930 98±1 2±1 3 5/2⁻
412 100 1 1/2⁻
1 3/2⁻
^{55}Fe

Figure 4



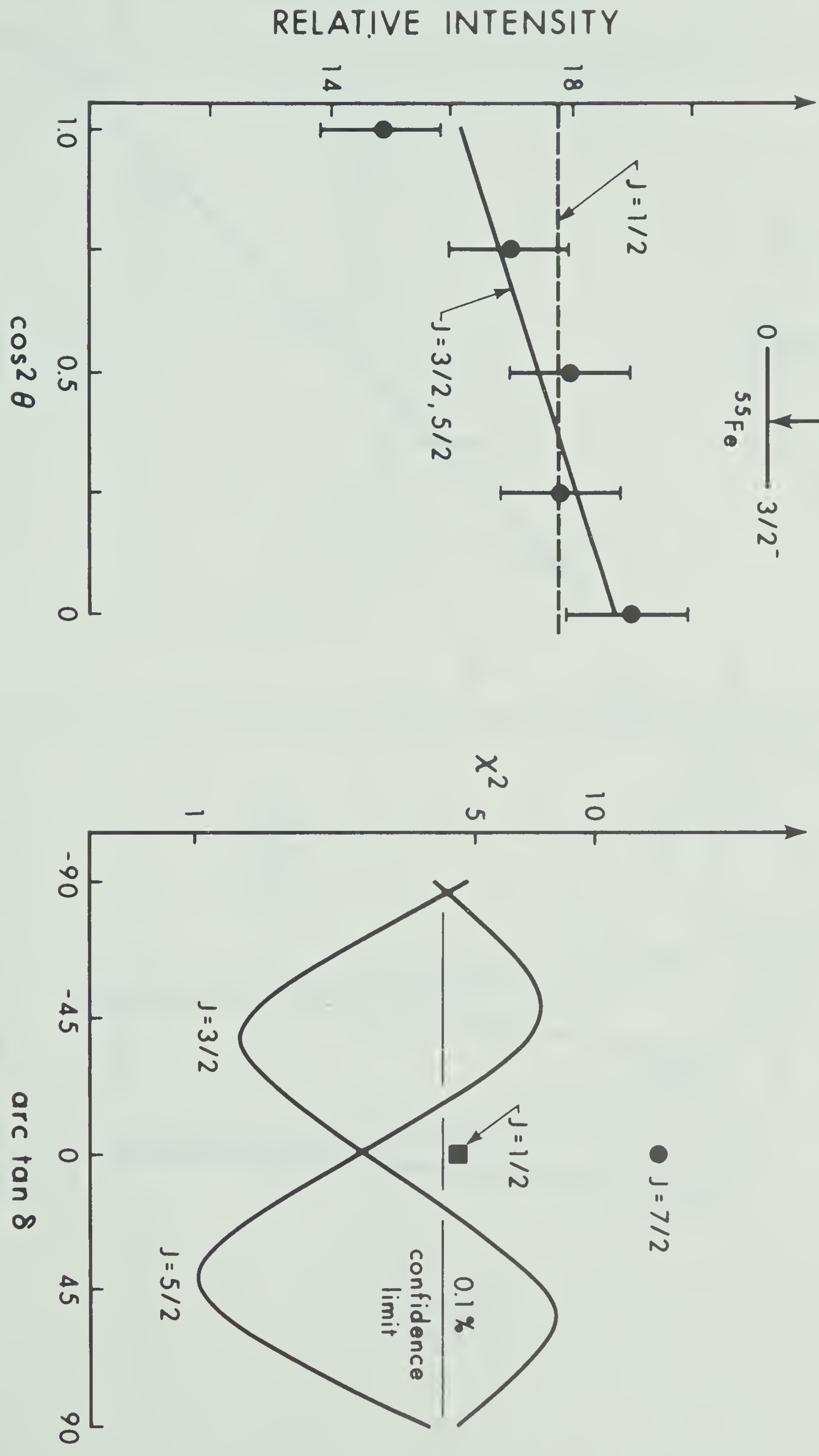
RELATIVE INTENSITY
16
14
1.0
0.5
0
cos2θ
J=7/2
J=3/2
1316
0
J
3/2⁻
55Fe
χ2
50
10
5
1
-90
-45
0
45
90
arc tan δ
0.1%
confidence
limit
J=5/2
J=3/2
J=7/2

Figure 4 a



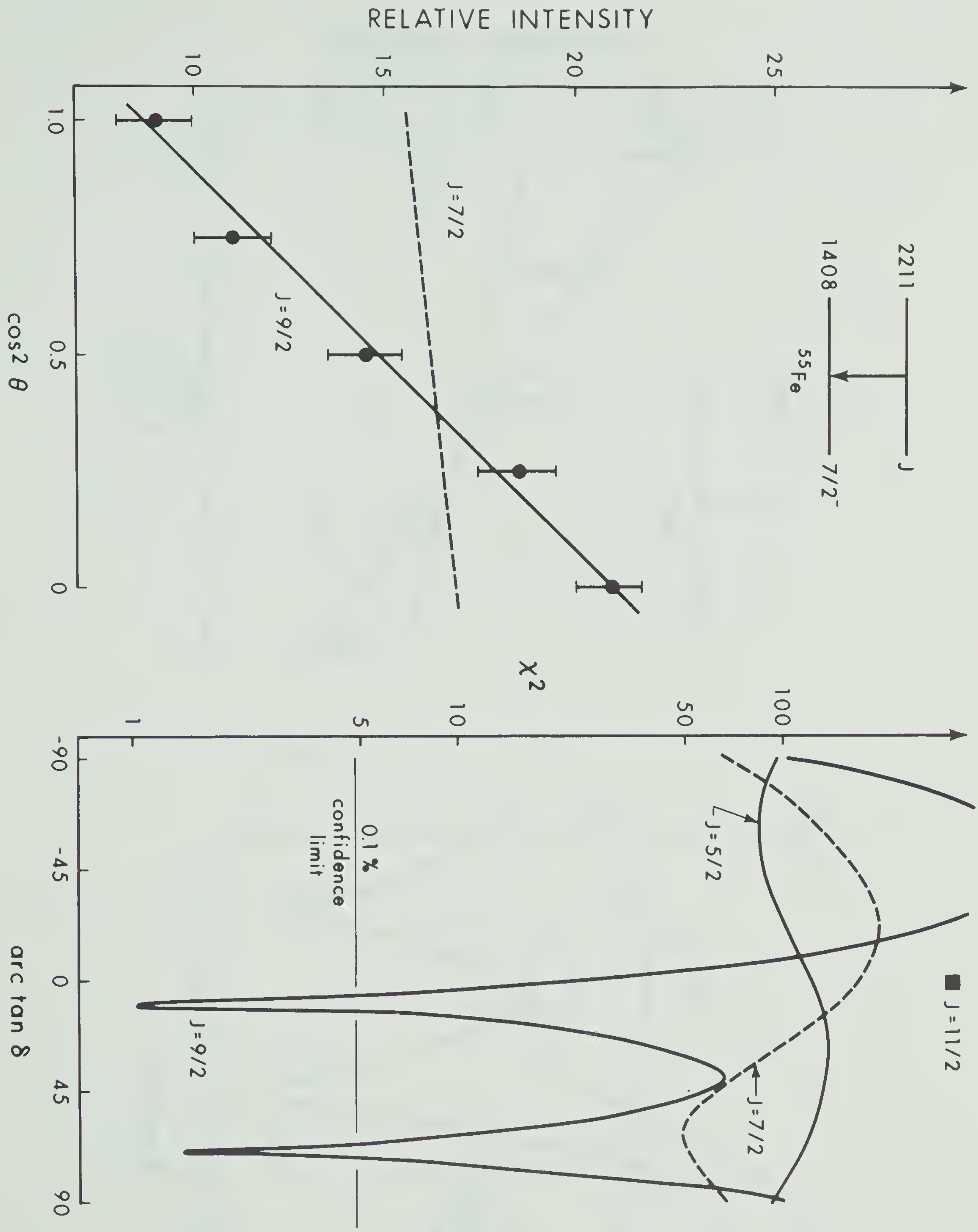
RELATIVE INTENSITY
30
34
38
1.0
0.5
0
$\cos^2 \theta$
J=7/2
J=5/2
1408
0
J
$3/2^-$
^{55}Fe
χ^2
1
5
10
50
-90
-45
0
45
90
arc tan δ
J=3/2
J=5/2
J=7/2
0.1%
confidence
limit

Figure 4b



RELATIVE INTENSITY
14
18
1.0
0.5
0
cos² θ
J=1/2
J=3/2, 5/2
2144
0
J
3/2⁻
55Fe
χ²
1
5
10
-90
-45
0
45
90
arc tan δ
J=3/2
J=5/2
J=1/2
J=7/2
0.1%
confidence
limit

Figure 4c



RELATIVE INTENSITY
10
15
20
25
1.0
0.5
0
cos2 θ
J=7/2
J=9/2
2211
1408
55Fe
J
7/2-
χ2
100
50
10
5
1
-90
-45
0
45
90
arc tan δ
J=5/2
J=7/2
J=9/2
J=11/2
0.1 %
confidence
limit

Figure 4d

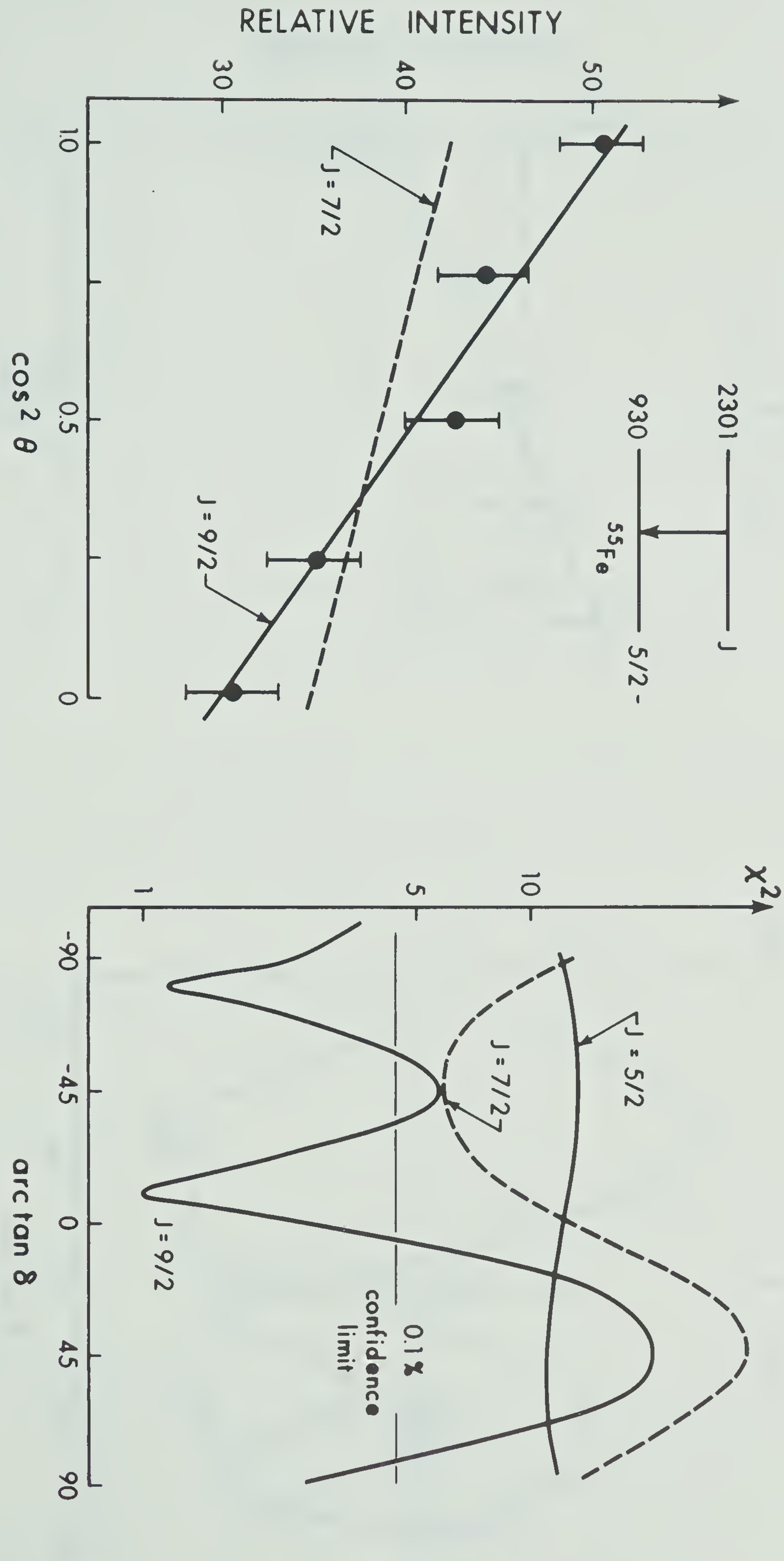
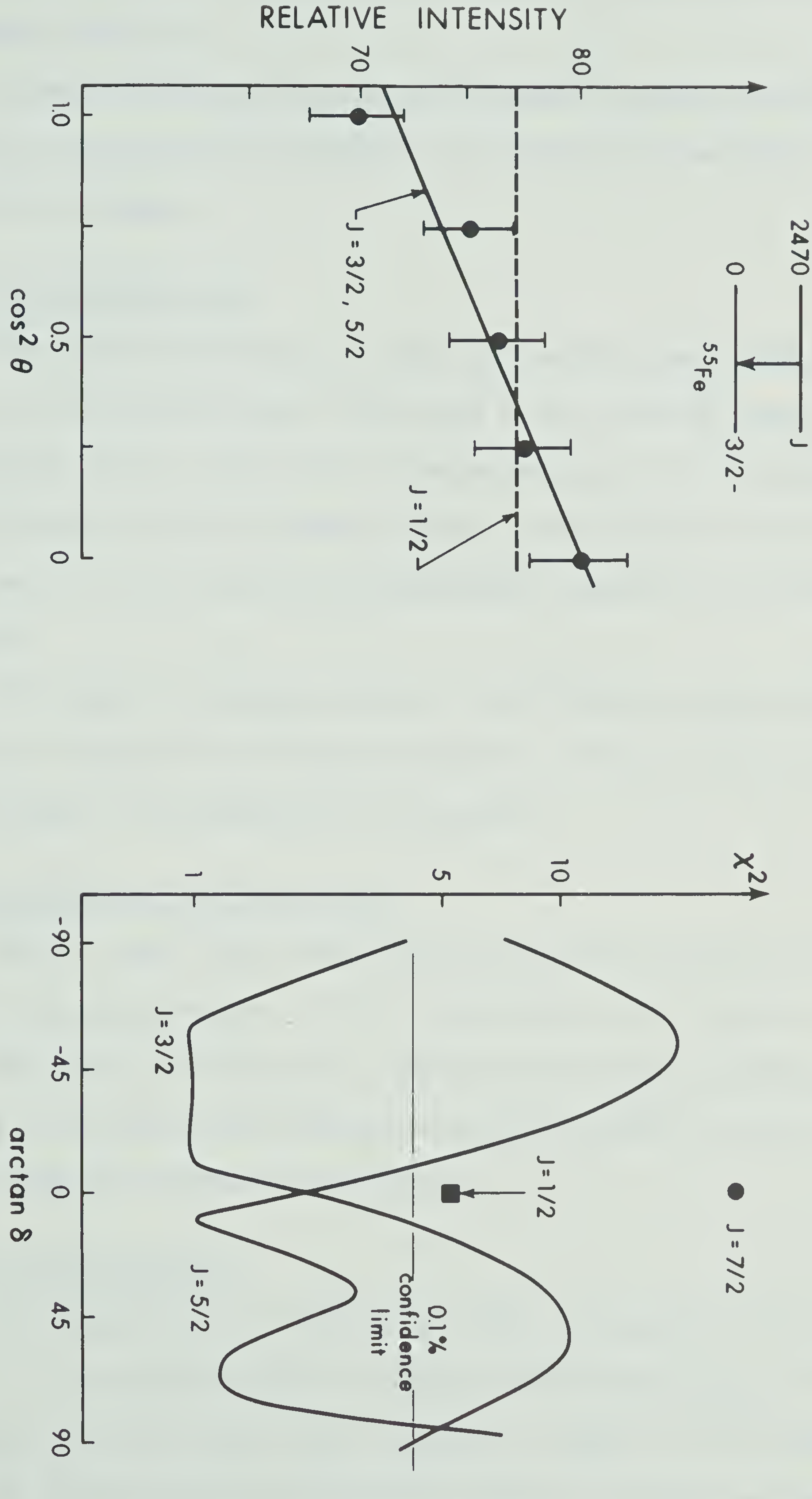

RELATIVE INTENSITY
50
40
30
1.0
0.5
0
$\cos^2 \theta$
J = 7/2
J = 9/2
2301
930
J
5/2 -
55Fe
χ^2
10
5
1
-90
-45
0
45
90
arc tan δ
J = 5/2
J = 7/2
J = 9/2
0.1%
confidence
limit

Figure 4e



RELATIVE INTENSITY
80
70
1.0
0.5
0
$\cos^2\theta$
2470
0
^{55}Fe
J
3/2-
J = 3/2, 5/2
J = 1/2
χ^2
10
5
1
-90
-45
0
45
90
arctan δ
J = 7/2
J = 1/2
J = 3/2
J = 5/2
0.1%
confidence
limit

Figure 4f

that the χ^2 fit is consistent only with the J=7/2 assignment and pure quadrupole radiation.

Fulmer (Fu 63) has shown that this state is weakly excited in the ^{54}Fe(d,p) reaction and has assigned an ℓ_n-value of 3, consistent with the J=7/2 assignment.

3.7 The 1.408 MeV Level

This level was observed to decay 45% to the ground state and the rest to the 0.930 MeV state; a 10% branch to the 1.316 MeV level reported by Fischbeck (Fi 66) could not be observed because of its low energy. The 1408 keV transition is again anistropic (a_2=0.13±0.02) and the χ^2 plot shows the spin to be 7/2 with quadrupole radiation, as indicated in Figure 4b.

This state is strongly excited in the ^{56}Fe(p,d) reaction (Gi 66); much more so than in the ^{54}Fe(d,p) reaction. This has led to a tentative 7/2 assignment, in agreement with our results.

3.8 The 1.920 and 2.051 MeV Levels

Both of these levels decay only to the ground and first excited states, indicating low spin. This is confirmed by the isotropic ground state transitions of both levels, consistent with spin assignments 1/2, 3/2, and in agreement with the (d,p) work (Ma 64) which has led to ℓ_n=1 angular momentum transfer for both states.

3.9 The 2.144 MeV Level

The observed decay of this level (20% to ground and 1.316, 60% to 0.930) is in disagreement with the results of Fischbeck ét al. (Fi 66) who report only the ground state transition. However, as they themselves point out, these transitions occurred in regions of their spectrum where limits on their intensities were larger than the intensity of the ground

state transition.

Only the ground state transition was observed to be anisotropic (a_2=-0.11$\pm$0.03), consistent with both the 3/2 and 5/2 assignments. The (d,p) experiments previously mentioned indicate ℓ_n=3 for this level; this rules out the J=3/2 assignment. The angular distribution and χ^2 plots are shown in Figure 4c.

3.10 The 2.211 MeV Level

The primary decay is by an 803-keV gamma ray to the 1.408 level (J=7/2). A weak 895-keV gamma may be due to a transition to the 1.316 MeV level with intensity < 20% of the total decay. The lack of transitions to states of lower spin than 7/2 is indicative of a high spin state; this is confirmed by the angular distribution of the 803-keV gamma ray. It is markedly anisotropic and becomes more so as threshold energy is approached, in accord with the predictions of the statistical model (a_2=-0.30±0.05 at 200 keV above threshold; a_2=-0.54±0.05 at 100 keV above). The χ^2 plot in Figure 4d shows this is consistent only with the J=9/2 assignment. Possible mixing ratio solutions are $\delta=2.75^{+0.75}_{-0.50}$ and $\delta=0.19^{+0.07}_{-0.09}$.

3.11 The 2.301 MeV Level

The predominant decay is to the 0.930 MeV state; weak transitions to the 1.316 and 1.408 MeV levels were also observed. The latter transition unfortunately has the same energy as the 2.211→1.316 MeV decay and so may belong to either one. The angular distribution of the major decay is again very anisotropic (a_2=0.39±0.05) and is consistent as shown in Figure 4e only with the J=9/2 assignment and pure quadrupole radiation. It is interesting to note that the E2 transition to the 5/2-state dominates the weak transitions to the 7/2-state.

3.12 The 2.470 MeV Level

The slightly anisotropic distribution of the ground state transition (a_2=0.05±0.02) is consistent with both J=3/2 and 5/2, but not with J=1/2, as shown in Figure 4f. Together with the ℓ_n-value of 1 as determined from the (d,p) reaction this implies that the spin of this state is $3/2^-$, in agreement with the ($n\gamma\gamma$) correlation measurements (Co 64). The mixing ratio was undetermined.

3.13 The 2.578 MeV Level

The only observed decay was to the ground state. The near isotropy of the gamma ray distribution (a_2=-0.05±0.04) precludes the spin assignment J=7/2. Although J=1/2 and 3/2 are allowed solutions on the basis of the χ^2, the stripping reactions have shown that this rather weakly excited state has a proton angular distribution strongly peaked near 40°, from which the authors have concluded ℓ_n=3 or 4. Assuming ℓ_n=3 (the value 4 is in disagreement with the β decay data of Fischbeck) the spin of this state is probably $5/2^-$.

3.14 The 2.877 and 2.950 MeV Levels

These levels are quite weakly populated in the ^{55}Mn(p,n) reaction. Angular distributions of the ground state transitions taken at 4.25 MeV were isotropic, limiting the spins of both states to J≤7/2. Statistics were poor for both gamma rays due to the low yield and the errors correspondingly large, limiting the information which could be extracted. The levels are also weakly populated in the ^{54}Fe(d,p) reaction and the proton distributions are structureless. Glashausser and Rickey (Gl 66) have shown that the "2.90 MeV state" is excited strongly in the (p,d) reaction and have suggested a spin of 7/2 to this state. To confirm this result, the gamma ray angular distribution should be re-examined using larger detectors and longer runs.

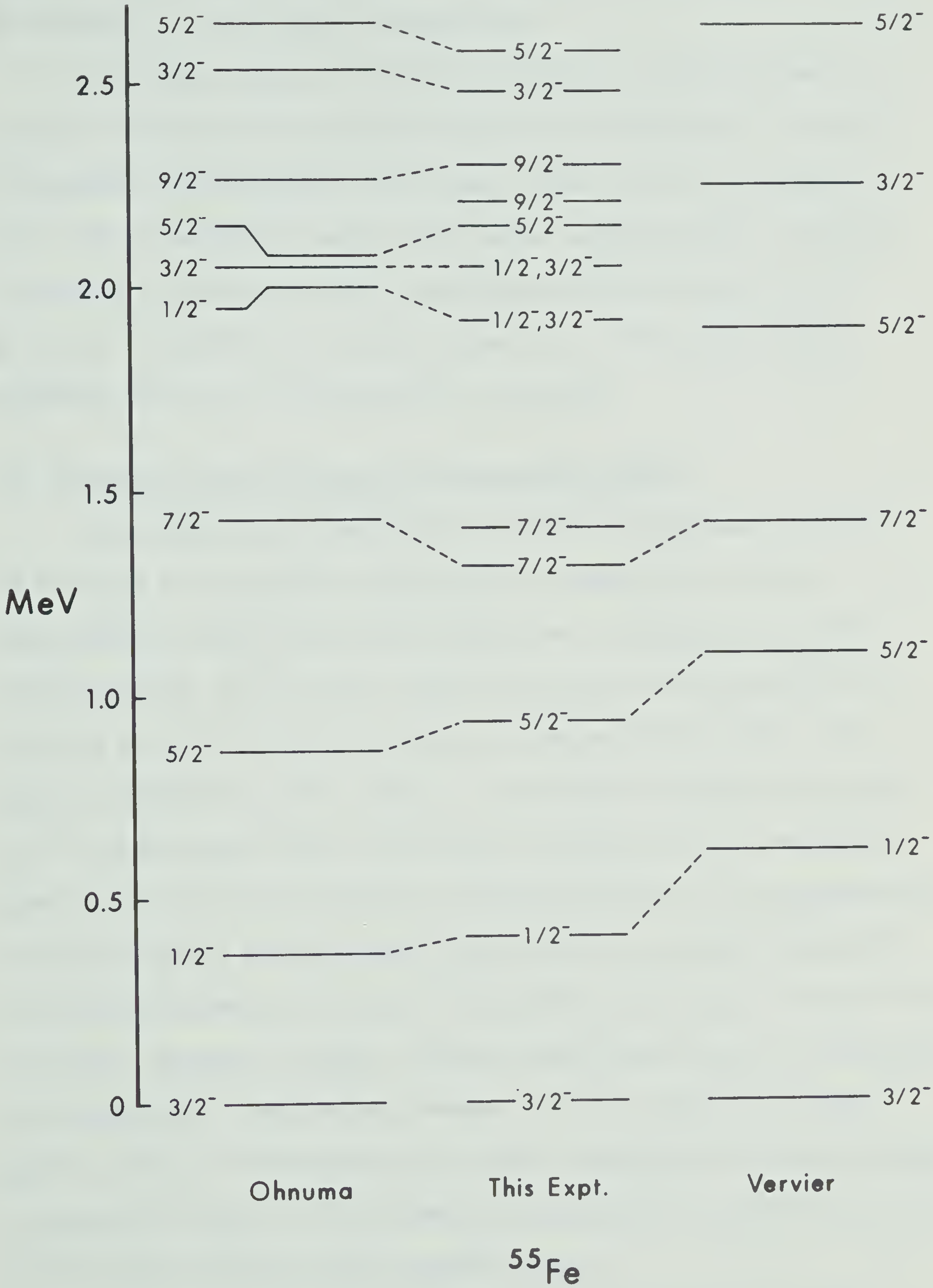
CHAPTER 4

DISCUSSION

4.1 Comparison with the Shell Model

Figure 5 shows the experimental level scheme and spins in ^{55}Fe as deduced here together with the results of two shell-model calculations, those of Ohnuma (Oh 66) and Vervier (Ve 66). It is evident that Ohnuma's results agree extremely well with the experimental data. The spins and excitations of the low-lying states are excellently reproduced, with the exception of one of the observed 7/2 states near 1.4 MeV. As the 1.408 MeV level, populated so strongly in the (p,d) reaction, is probably the $f_{7/2}^{-1}$ hole state, we would not expect it to be predicted by a calculation considering the effect of only particles (and not holes). Thus the 7/2- level predicted in both calculations is very likely the 1.316 MeV state. Quite impressive also is the fact that Ohnuma's predictions are equally good at much higher excitations—the predicted triplet at ~2 MeV of spins $1/2^-$, $3/2^-$, and $5/2^-$ and two 9/2 states near 2.2 MeV have all been observed in this experiment. Ohnuma however predicts only one $9/2^-$ state. One of the experimental 9/2 states decays predominantly to the 1.408 MeV hole state; accordingly, the former is probably described by a wave function with considerable hole admixture and would not be predicted either in a shell-model calculation. The other 9/2 state at 2.301 MeV decays by a strongly enhanced E2 transition to the 5/2- state at 0.930 MeV and is probably the one predicted by Ohnuma. Because both levels decay by strongly anisotropic gammas, a polarization experiment to determine parities is both possible and interesting—such an experiment is contemplated. In addition, Ohnuma predicts an $11/2^-$ and a $13/2^-$ level between 2.5 and 3.0 MeV. Levels of such high spin would be weakly excited due to the large centrifugal barrier; possible transitions from weak stripping

Figure 5. *Energy levels and spins in ^{55}Fe as predicted by shell model calculations of Ohnuma and Vervier are compared with the results obtained from the present experiment.*



MeV
2.5
2.0
1.5
1.0
0.5
0
5/2⁻
3/2⁻
9/2⁻
5/2⁻
3/2⁻
1/2⁻
7/2⁻
5/2⁻
1/2⁻
3/2⁻
5/2⁻
3/2⁻
9/2⁻
9/2⁻
5/2⁻
1/2⁻,3/2⁻
1/2⁻,3/2⁻
7/2⁻
7/2⁻
5/2⁻
1/2⁻
3/2⁻
5/2⁻
3/2⁻
5/2⁻
7/2⁻
5/2⁻
1/2⁻
3/2⁻
Ohnuma
This Expt.
Vervier
^{55}Fe

Figure 5

states near 2.6 and 2.8 MeV reported by Sperduto (Sp 61) were not observed in the present experiment. A search for them, using a large GeLi detector, is also under contemplation.

In contrast, Vervier's results are not in as good agreement. Only the first three excited states have predicted energies and spins in agreement with experiment; the higher levels are not reproduced at all. This is presumably due to the greater approximations introduced by Vervier, in particular, his consideration only of levels of spin up to 7/2. The effect of using an unrealistic (δ-function) radial dependence for the n-p interaction is not known.

4.2 Comparison with the Particle-Core Coupling Model

As pointed out by Ramavataram (Ra 63) and Thankappan and True (Th 64) both single-particle and collective behaviour in the low-lying states of medium heavy odd-A nuclei are reproduced by coupling the odd particle to an excited state of the even core, especially if the extra particle is the only one in a single-particle state. The simplest prediction of this model is the theorem of Lawson and Uretsky (La 57) which predicts that the "centre of gravity" of the multiplet formed by all possible couplings of the odd particle to an excited state of the core (using the statistical factor 2J+1 as a weight) should lie at an excitation equal to that of the excited core state. No prediction of the spin sequence is made; a suitable Hamiltonian must be constructed and diagonalized. The theorem, moreover, is only valid in the weak coupling limit; if interactions are strong, considerable mixing of states is expected to occur and the strength of a particular configuration may be spread over several widely separated levels.

Thankappan and True have calculated the levels of some Z=29 nuclei (the copper isotopes) which are similar in structure to the N=29

nuclei, using the generalization suggested by de-Shalit (Sh 61) and found quite good agreement below 1.5 MeV. The simpler model, assuming weak coupling and widely separated single-particle states, did not give as good agreement; this suggests the latter assumptions are not justified. It appears probable that the agreement (especially at higher energies) would have been improved by including more excited states of the core in the calculations. Earlier, Ramavataram (Ra 63) used a simpler model to predict the states of ^{55}Fe. Having assumed the validity of weak coupling, his results are not in as good agreement with the experimental data as the shell-model calculations. The generalized approach, however, is clearly not without merit and calculations similar to Thankappan's applied to ^{55}Fe are under consideration.

Another interesting consequence of this model is that each member of the multiplet should decay predominantly to the ground state (corresponding to the decay of the excited state of the core). Furthermore, the value of B(E2) of each transition should be the same as for the core state. Most even nuclei in the mass region under consideration here have a collective 2+ first excited state and hence enhanced E2 ground state transitions. Levels in the neighbouring odd-A nuclei formed by particle coupling to these states should also decay predominantly by E2 radiation to the ground state. If the first 1/2- state in ^{55}Fe (at 0.412 MeV) can be so described, the M1 radiation should be heavily retarded and the lifetime long, of the order of 1 nsec on the single particle estimate. The structurally similar isotopes ^{63,65}Cu reportedly do show large values of the B(E2) in agreement with the theory (Th 64); one may assume a similar occurrence in ^{55}Fe. We may indeed find four states in this nucleus of the proper spin and with the appropriate decay properties; the 1/2- at 0.412, the 5/2- at 0.930, the 7/2- at 1.316, and

[illegible]

the (possibly) 3/2- at 1.917 MeV. The centre of gravity of this set of levels lies at 1.25 MeV while the first excited state of ^{54}Fe is at 1.4 MeV. A strict proof of this admitted conjecture will require some estimates of B(E2)'s and lifetime measurements. It is likely also that the strength of one or more of these states (for example, the 3/2-) may be split over several levels.

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APPENDIX I

THE COMPOUND NUCLEAR STATISTICAL THEORY OF GAMMA-RAY ANGULAR DISTRIBUTIONS

THE COMPOUND NUCLEAR STATISTICAL THEORY OF GAMMA RAY ANGULAR DISTRIBUTIONS

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UAE-NPL-14

Internal Report

September, 1969

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1. Introduction

A frequent approach for determining the spin of nuclear energy levels is by the study of γ-ray angular distributions and correlations following nuclear reactions. The angular distribution of a γ-ray transition from a state of spin J_2 to one of J_3 is given by (Li 64)

$$W(\theta) = \sum_{m,m'} P(m)|\langle J_2 m|J_3 m' L\ m-m'\rangle|^2 F_L^{m-m'}(\theta) \qquad 1)$$

where P(m) is the relative population of the m'th magnetic substate (recalling that a state of spin J has 2J + 1 magnetic substates or possible orientations with respect to the axis of quantisation), $\langle J_2 m|J_3 m' L\ m-m'\rangle$ is a Clebsch-Gordan coefficient and $F_L^{m-m'}(\theta)$ is the pattern for radiation of multipolarity L and magnetic quantum number change m-m'. For example, dipole radiation (L=1) is characterised by a $\sin^2\theta$ term; the origin of this is formally the same as for the well-known result that a (classical) oscillating dipole emits no radiation in the direction of the dipole axis. The exact forms of the functions

$F_L^{m-m'}(\theta)$ may be obtained from the usual multipole expansions of the electromagnetic field; they are related to the vector spherical harmonics and are essentially the functions $X(\theta,\phi)$ tabulated by Jackson (Ja 62), page 551.

The angular distribution of radiation from a state of given spin is seen to depend on the multipolarity L (where L satisfies the usual triangle relation, $|J_2-J_3| \leq L \leq J_2+J_3$) and the population parameters P(m). The latter functions are very instrumental in determining the extent of the anisotropy. If the magnetic substates are equally populated; i.e. each $P(m) = \frac{1}{2J+1}$ one may show (e.g. from the Clebsch-Gordan series and the addition rule for spherical harmonics) that the distribution $W(\theta)$ becomes isotropic. Physically, this is because all possible orientations of the spin axis are equally likely, and as no particular spatial direction has been selected, the gamma ray is emitted in any direction with equal probability. Such a case is clearly not very useful as it gives us no information on either the level spin or the nature of the decay radiation.

To extract this information, an alignment of the nucleus is necessary. An aligned nuclear state is one for which P(m) = P(-m) for a particular substate m, but with different populations for the various values of |m|. For a true alignment to occur (equal populations for positive and negative values of a given m) all radiations leading to the aligned state must be unpolarized. For example, if the state is formed by proton capture, the two spin orientations $m_s = \pm 1/2$ must be equally probable. Otherwise we are led to states with $P(m) \neq P(-m)$; such a state is said to be polarized rather than aligned. These will not be discussed in the sequel.

To obtain alignment, therefore, a nuclear reaction populating only the lower magnetic substates (in absolute value!) must be selected. There are several possible ways of achieving this:

i) by radiative capture of α-particles or protons on a low spin target; the bombarding particles themselves have low spin and their orbital angular momenta, being perpendicular to the beam direction (quantisation axis), will have zero component along this axis and, therefore, will not contribute to the de-alignment of the final state;

ii) Method II of Litherland and Ferguson (Li 61) in which γ-rays are detected in coincidence with charged particles at 0 or 180 degrees, the latter condition ensuring that no component of the <u>outgoing</u> orbital angular momentum along the quantisation axis contributes;

iii) nuclear reactions close to threshold for production of the level; if the outgoing particles have low energy, then s-waves ($\ell = 0$) will predominate. This last method is the one of interest in this report.

As a particular example of method iii), consider the (p,nγ) reaction on a target of relatively low spin. The total angular momentum $j_1 = \ell_1 + s_1$ of the incoming proton may be combined vectorially with the target spin J_0 to form a final state of spin and parity restricted only by the relevant triangle inequalities which must always be satisfied. For sufficiently large ℓ_1, there are a large number of ways in which the combinations may be made; most of them will lead to states of low or intermediate spin, but for a few cases, if the momenta happen to add numerically, a high spin state will result in the compound nucleus (CN).

This state will be strongly aligned since the only contributions to the P(m)'s of the CN state are from the target spin and s_1 (the intrinsic spin of the bombarding particle); ℓ_1 will not contribute because it is perpendicular to the axis of quantisation. The aligned CN state may then decay to a state of lower excitation by emitting a nucleon (or rarely, a γ-ray). If a nucleon is emitted, its total spin will be $j_2 = \ell_2+s_2$ and will tend to de-align the CN state (that is, populate additional magnetic substates in the residual level). As it may be emitted in <u>any</u> direction (if it is not "observed", that is, if no coincidence measurements are made), the projection of ℓ_2 will no longer be zero and the residual state will be only weakly aligned. If however the reaction proceeds near threshold for production of the final state, the energy of the emitted nucleon will be low and $\ell_2 = 0$ partial waves will predominate; thus $m_{\ell_2} = 0$ also and the de-alignment will be minimized. (Usually, the $\ell_2 \geq 1$ admixture in the total outgoing wave is small; in certain cases it must be considered as well. This leads to a small reduction of the alignment and hence in the anisotropy of the emitted γ-ray.) The net result is that in the residual nuclear state, the only appreciably populated magnetic substates are those up to $|m| = J_0$. If $J_2 > J_0$, the state will be aligned (provided $\ell_2 \cong 0$) and anisotropic γ-ray distributions are to be expected. Since the anisotropy is dependent on both J_2 and J_3 (the level to which state J_2 decays), the spin J_3 must usually be known to permit a unique determination of J_2, or vice versa.

2. Assumptions of the Statistical Model

Reactions of the form $(p,n\gamma)$ and $(\alpha,n\gamma)$ proceed primarily by the compound reaction mechanism. As the Q-values for capture of protons and alpha-particles are normally very large (> 7 MeV), these reactions for medium-weight nuclei pass through an intermediate state where the level density is high (> 5 levels per keV). If a thick target is used, the reaction proceeds via a large number of 'overlapping levels' in the compound nucleus, and the statistical model is justified. The general theory of statistical reactions and the S-matrix formalism is given in (Bl 52, Er 63). The essence of their argument relevant to our case is that certain interference terms arising in the distribution formulae between different CN spins average out to zero if a large number of these intermediate states are involved. Ericson (Er 63) shows that these interference terms normally lead to fluctuations in the cross section of the order of $1/(nN)^{\frac{1}{2}}$, where

N is the number of overlapping levels involved

n is the number of residual levels to which each of the CN levels may decay.

In $(p,n\gamma)$ reactions on medium weight nuclei, N is very large, especially with a thick target. Although n is much smaller, the fluctuations nevertheless become unobservable. The interference terms may therefore be dropped. Furthermore, the use of a thick target permits one to pass through a large number of CN levels, increasing the probability of populating high spin levels in the residual nucleus.

We now proceed to derive a formula for the gamma ray angular distribution following a compound nuclear reaction with the assumptions of the statistical model. Essentially, we must reduce the unknown P(m)'s of (1) into products of vector-coupling coefficients and reaction-dependent functions ('transmission coefficients'). This formula reduces to a similar equation derived by Sheldon (Sh 66).

3. Calculation of the Angular Distribution

We use the same notation as described previously. Define further the total incoming and outgoing angular momenta by $j_1 = \ell_1 + s_1$ and $j_2 = \ell_2 + s_2$. In all cases primes will denote interfering amplitudes. Goldfarb's generalized correlation formula (Go 59, eqn. 7.9) is

$$W(\theta) \sim \sum A_k(j_1 j_1' J_0 J_1 J') \, A_k(J_1 J_1' J_2 J_2' j_2 j_2') \times A_k(LL' J_3 J_2 J_2') \qquad (2)$$

$$\{ \mathrm{Re} \mid < J_3 L J_2 j_2 J_1 \, |\theta| \, J_0 j_1 J_1 > \times < J_3 L' J_2 j_2' J_1' \, |\theta| \, J_0 J_1' J_2' > \mid \} P_k(\cos\theta).$$

The summation is over $j_1 j_1' J_1 J_1' j_2 j_2' J_2 J_2'$ (although some of these can be omitted, as will be shown presently). The A_k coefficients have the form

$$A_k = (-)^p \times \text{Clebsch-Gordan} \times \text{Racah}$$

where the parity of p depends on the spins involved. The physical significance of these (as with all vector-coupling) coefficients in the angular distribution formalism is as follows. Transitions (either radiative or by capture of emission of particles) between two states of non-zero spin are

considered to take place between the individual magnetic substates of the levels. (These "microtransitions" cannot, of course, be observed experimentally in nuclear physics since their separation is much smaller than the natural width of most nuclear levels.) The relative intensities of these transitions are not equally probable, but are proportional (as can be shown from the Wigner-Eckart theorem) to certain geometrical coefficients depending on the spins J and projections m_j of the levels involved. For a gamma-ray transition between two levels, this reduces to the Clebsch-Gordan coefficient since for a photon $m_L = \pm 1$ always (where L is the multipolarity) and only the spins of the two levels arise. For transitions induced by particle decay or absorption, where the particle may have intrinsic angular momentum as well as arbitrary projection of the orbital angular momentum, Racah coefficients or 6-j symbols (describing coupling of 3 angular momenta) are to be used. Each particular gamma-ray micro-transition between a pair of magnetic substates has its own characteristic radiation pattern (the function $F_L^{m-m'}(\theta)$ described earlier) and the total correlation following a number of transitions between levels, whether observed or not, will be a rather messy product of all these coefficients and functions. This is the essential meaning of the geometrical factors of (2). (The $F_L^{m-m'}(\theta)$ function is proportional to the associated Legendre polynomials which can in turn be reduced to the ordinary Legendre polynomials used in (2).)

The sum over all the indices is necessary because of all the possible ways in which a state of spin J_0 can eventually lead to a state of spin J_3

following the absorption and emission of particles and gamma rays of arbitrary angular momenta.

This now takes care of everything except the terms Re |< >|. Up to now we have only considered the geometrical terms arising from the multitudinous ways in which a number of spins can couple to yield a desired final state. We must, however, also include terms describing the reaction mechanism--the actual physics of the reaction--and this we may do via the elements of the S-(or reaction) matrix. These are also dependent on the angular momenta as well as the energies involved and are heavily dependent on the reaction mechanism. Our knowledge of nuclear forces is at present not sufficiently accurate for the S-matrix to be exactly known. We must, therefore, assume some sort of simple model, which may or may not be realistic, on which basis we may evaluate the matrix elements. The test of the model, as with all models, consists in comparing experimental data with the predictions of the model. Usually (as in reactions proceeding through isolated resonances and especially in stripping reactions) the matrix elements are hard to calculate. We shall show that using the assumptions of the statistical model, in which the reaction passes through a large number of overlapping levels, the matrix elements can be replaced by energy-averaged Transmission Coefficients (TC), which can be obtained from a number of computer programmes or tabulations.

We note that in Eqn. 2 the sum is taken coherently over sets of interfering angular momenta. This arises for the incoming and outgoing waves because (Bl 52a) each partial wave has associated with it a definite

phase shift δ_ℓ which depends on the form of the potential chosen. There is thus a definite phase relationship between the various partial waves and the contributions from each must be added together coherently. This means that the contributions from each partial wave cannot be treated separately but must be considered together.

Interference terms also arise among states of the compound nucleus. At high excitation energies, the lifetime τ of the compound nucleus is small and the level width $\Gamma = \hbar/\tau$ correspondingly large. Eventually, we may reach a state where Γ is so large that a number of levels is contained within it and each level makes its influence felt over a large energy range including many other levels. This happens when $\Gamma >> D$ where D is the level separation. We must now consider transitions simultaneously over all the states within an energy range Γ and take the interference terms into account. Although Γ may be regarded as a width of the compound nucleus, it is more appropriate to regard it as a "coherence energy" within which it is necessary to treat matrix elements between the various overlapping levels as coherent, and introducing interference. Mathematically, the interferences arise because of the assumption that the phases of the matrix elements are randomly distributed. The sum of a large number of elements of random phases is a normally distributed random number. As the cross section is written as a squared modulus of a sum over probability amplitudes (matrix elements), this will also be random. We therefore expect strong fluctuations (Ericson fluctuations) in the cross section. A more "physical" reason for the interferences is that any two levels separated

by less than Γ will partially overlap. Because of the phase relations between the wavefunctions describing these levels, they will interfere where they overlap. A crude analogy may be set up with interference between waves of different phases in optics. The result is that at slightly different energies the mechanism leading to the final state and hence the properties (such as population parameters) may be markedly different.

However, if the reaction proceeds through a prodigious number of levels over an energy range large compared to Γ (as we have if we use a thick target or a beam with poor energy resolution),then we may invoke statistical considerations. Now the cross section will contain contributions over several ranges of coherence energy. Although the contributions from each particular range of Γ are random and fluctuate wildly, we may add them together incoherently because levels separated by much more than Γ cannot interfere. The resulting energy-averaged cross section will not fluctuate because the average of several sets of random numbers of the same distribution will have a much smaller variance than that of the individual numbers.

In the statistical model, then, the levels of the compound nucleus do not interfere and the sum over J_1' may be dropped.

We can also show that there will be no interference terms arising from the various outgoing partial waves. The outgoing particles are, under our assumptions, unobserved; the interference terms must, therefore,

be averaged over the entire sphere and this can be shown (see, e.g., Blatt and Weisskopf (Bl 52a)) to vanish.

We remain with the interference terms over the incoming partial waves $j_1 j_1'$. Again Blatt and Weisskopf assert that in the statistical hypothesis these terms vanish, but give no proof. The following explanation is due to Dotsenko (Do 69).

A generalized angular distribution expression contains contributions from terms

$$I = \int \psi(\underline{r})\ V\ (\underline{r},\underline{r}')\ \psi\ (\underline{r}')\ d\Omega \qquad (*)$$

where $\psi(\underline{r})$ can be separated into contributions due to radial, angular and spin-dependent components; generally, we may write

$$\psi(\underline{r}) = R(r)\ Y_{\ell Js}^{m}\ (\theta,\phi).$$

The potential $V(\underline{r},\underline{r}')$ describes the interaction between the incoming and outgoing particles with the nucleus; this leads to our earlier assertion that the interference terms depend on the form of the interaction chosen. Generally, this potential is not separable into radial and angular-dependent functions; contributions from these must then be considered together and the entire integral contains terms in both ℓ and ℓ' which do not cancel, leading to interference. This occurrence is then a particular example of channel coupling. Now consider the following (justifiable!) simplification: let us suppose that spherical symmetry holds. Then we may write

$$V(\underline{r},\underline{r}') = V(|\underline{r}-\underline{r}'|)$$

that is, V depends only on the distance between the nucleon and particle and not on the direction. The integral (*) can then be rewritten

$$I = \int R(\underline{r})\ V(|\underline{r}-\underline{r}'|)\ R(\underline{r}')\ d^3\underline{r}' \int Y^m_{\ell Js}(\theta\phi)\ Y^m_{\ell' Js}(\theta\phi)\ \sin\theta d\theta d\phi$$

But the spherical harmonics $Y^m_{\ell Js}$ are orthogonal functions; hence

$$\int Y^m_{\ell Js}\ Y^m_{\ell' Js}\ \sin\theta d\theta d\phi = \frac{2}{2\ell+1}\ \delta_{\ell\ell'}$$

and contributions to the angular distributions arising from interference between partial waves vanish.

Each term in the sum now conserves total angular momentum and the elements of the S-matrix may be factored into products of reduced matrix elements and Clebsch-Gordan coefficients via the Wigner-Eckart theorem. The correlation becomes

$$W(\theta) = \frac{1}{\hat{S}_1^2 \hat{J}_0^2} \sum A_k\ (j_1 J_0 J_1)\ A_k\ (j_2 J_1 J_2)\ A_k\ (LL' J_3 J_2 J_2)$$

$$\times < J_3 || L || J_2 > < J_2 || L' || J_3 >$$

$$\times |< J_2 || j_2 || J_1 >|^2\ |< J_1 || j_1 || J_0 >|^2$$

$$\times P_k\ (\cos\theta). \qquad (3)$$

The A_k coefficients in the above expression may be written in terms of the

Clebsch-Gordan and Racah coefficients using various identities in Goldfarb's article. Then Eqn. 3 may be written

$$W(\theta) = \frac{1}{\hat{S}_1^2\hat{J}_0^2} \sum^{1} (-)^{J_0+J_2-j_2-s_2} \hat{\ell}_1^2\hat{j}_1^2\hat{J}_1^4 (\ell_1 0 \ell_1 0|k0)$$

$$\times W(\ell_1\ell_1 j_1 j_1;ks)\, W(J_1 J_1 j_1 j_1;kJ_0)\, W(J_1 J_1 J_2 J_2;kj_2)$$

$$\times |< J_2||j_2||J_1 >|^2 \; |< J_1||j_1||J_0 >|^2$$

$$\times \frac{1}{1+\delta^2} \{ F_k(LLJ_3J_2) + 2\delta F_k(LL'J_3J_2) + \delta^2 F_k(L'L'J_3J_2)\}$$

$$\times P_k (\cos\theta).$$

The F_k have been tabulated by Ferenz and Rosenzweig (Fe 52) and δ is the mixing ratio of the γ-transition, defined by

$$\delta = \left| \frac{< \psi_f|L+1|\psi_i >}{< \Psi_f| L |\Psi_i >} \right|$$

where L and L+1 are the corresponding multipole operators. Thus, δ is the ratio of the amplitude of 2^{L+1} multipole radiation to the amplitude of radiation of order 2^L. For a transition between two levels of spins J and J', say, the possible multipolarities are $|J-J'| \leq L \leq J+J'$; of these, however, only the lowest two are of any importance. The parity change determines whether the radiation is electric or magnetic; the selection rules are

E0		M0	not allowed
E1	yes	M1	no
E2	no	M2	yes
E3	yes	M3	no

where 'yes' and 'no' refer to a change in parity and no parity change respectively. The transition probabilities diminish rapidly as one considers increasing multipole orders; further, for a given multipole, magnetic radiation is far less probable (1%) than the corresponding electric radiation. The result is that E2/M1 or E3/M2 mixing ratios may occur, but one almost never has reason to consider M2/E1 or M3/E2 competition.

All the terms in (4) except the reduced matrix elements are purely geometric and are readily evaluated. The latter, however, are reaction dependent and can be found only from a particular model. The statistical model will now be used to evaluate the matrix elements in terms of quantities known as Transmission Coefficients. These are related to the cross-sections for formation and decay of the compound nucleus. (For an explicit formulation, see Preston, Chapter 16 (Pr 62)). The spin-dependent cross-section for formation of the compound nucleus in a particular macrochannel α (a macrochannel is characterised by the incoming particle and target nucleus or, because of detailed balance, by the reaction products in the case of a decay) is

$$\sigma_{\alpha}^{\ell}\ (CN) \;=\; \pi\, \lambda\!\!\!^{-}{}_{\alpha}^{2}\ (2\ell+1)T_{\ell\alpha}$$

Here, $T_{\ell\alpha}$ is the transmission coefficient for an incoming particle of the ℓth partial wave. They are probabilities that the two particles of the entrance channel may combine to form a compound state. The probability of decay by a particular microchannel α' (characterized by the kind and angular momentum of the reaction products) through CN state J1 is

$$\sigma_{\alpha'} = \sigma_{cs}^{J1} \frac{T_{\alpha'}^{J1}}{\sum_{\alpha''} T_{\alpha''}^{J1}} \tag{6}$$

In other words, the CN decays in a given exit channel α' with a probability of the ratio of its transmission coefficient to the total TC for all possible exit channels. Thus, we assume implicitly that formation and decay of the CN are independent processes. The inherent assumptions are:

(1) The CN is a long-lived state
(2) A large number of states are involved in a small energy interval with randomly distributed widths and energies. (Statistical model).

The transmission coefficients for decay of a CN state have an intuitive meaning. We can write:

$$T = \frac{\text{number of emissions per second in channel } \alpha'}{\text{number of times per second that channel } \alpha' \text{ occurs}}$$

Since the single-particle probability for tunneling through a potential barrier is just the number of particles emitted per second, if there is

always a particle inside the well, $T_{\ell\alpha}$ may be identified with the s.p. probability for escape from the relevant potential. The TC may be obtained, for example, from a Hauser-Feshbach programme (Da 69) or the tabulations of Auerbach (Au 64).

We may now, in our model, replace the matrix elements in (4) by the cross-section for formation and decay of the compound nucleus in terms of the TC. The correlation finally becomes

$$W(\theta) = \frac{1}{\hat{s}_1^2 \hat{J}_0^2} \sum (-)^{J_0+J_3-j_2-s_2} \hat{\ell}_1^2 \hat{j}_1^2 \hat{J}_1 \hat{J}_2$$

$$\times\ (\ell_1 o \ell_1 o | ko)\ W(\ell_1 \ell_1 j_1 j_1 ; ks)\ W(J_1 J_1 j_1 j_1 ; kJ_0)$$

$$\times\ W(J_1 J_1 J_2 J_2 ; kj_2)$$

$$\times\ \frac{1}{1+\delta^2} \{ F_k(LLJ_3J_2) - 2\delta F_k(LL'J_3J_2) + \delta^2 F_k(L'L'J_3J_2)\}$$

$$\times \left(\frac{T_{j_1}^{J_1}\ T_{j_2}^{J_2}}{\sum\limits_{j_2''} T_{j_2''}^{J_2}} \right) P_k\ (\cos\ \theta)\ .$$

and is determined solely by the target spin J_0, the transmission coefficients, and the spins of the residual nucleus J_2, J_3, and the mixing ratio. Hand calculations may be performed to calculate $W(\theta)$ for particular cases, but a computer programme is to be preferred because of the large number of terms in the sum. The programme MANDY written by Sheldon was obtained and used for this purpose. The remainder of this report concerns its use.

4. Description of the Program

MANDY is a Fortran-language level H program compatible with the University of Alberta IBM 360 computer, originally written by E. Sheldon. Since then it has undergone several modifications to make it more flexible and useful, especially if used in conjunction with the GRID SEARCH program described elsewhere.

It calculates angular distributions of particles and gamma rays following compound nuclear reactions, using the theory outlined in Section 3 from formulae of the form (3.7). As most of the work done has been on gamma rays, the output format will be discussed for this case.

The notation used in the description is given in Fig. 1.

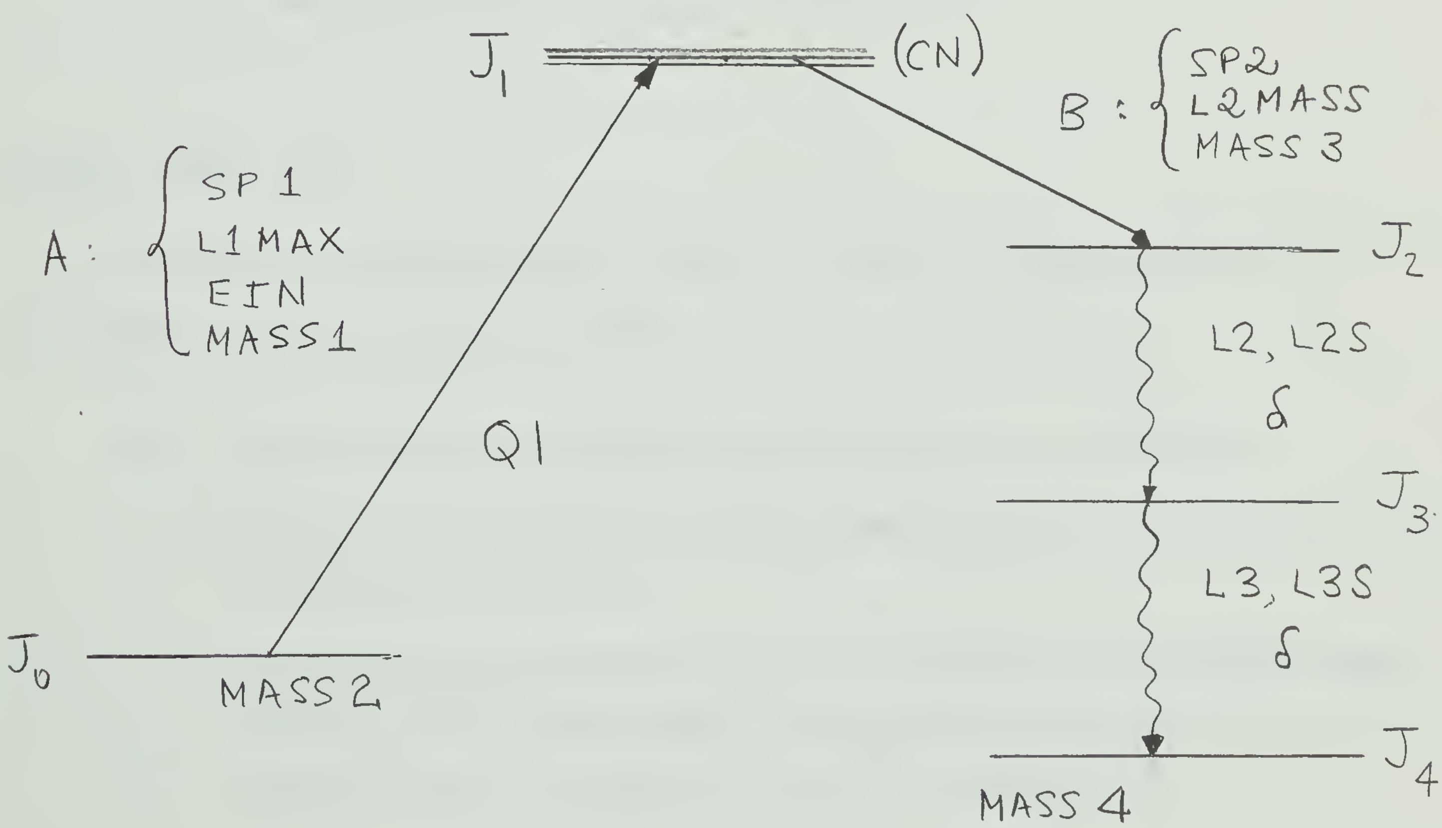

Fig 1.

where:

J0 is the spin of the target nucleus.

J2, J3, J4: spins of levels in the residual nucleus under consideration.

SP1 and SP2 are spins of incoming (A) and outgoing (B) particles.

MASS1, MASS2, MASS3, MASS4 are the masses of incoming, target, outgoing and residual nuclei (or particles) respectively.

L1MAX is the maximum orbital angular momentum of the incoming particle used in the calculations (usually ~ 5).

L2MAX, same but for outgoing particle (usually 2 - 3).

EIN is the beam energy in the laboratory system.

Q1 is the Q-value of the reaction.

EL is the excitation energy of level J2.

L2,L2S: multipolarities of decay of level J2.

L3,L3S: multipolarities of decay of level J3.

Card 1: Ø1 I3

Option Ø1 steers the calculation in a number of ways, depending on what is wanted, gamma or particle distribution, cascades, etc.

Ø1=1: Distribution of outgoing particle (B) is calculated.

2: Gamma distribution from (A,B-GAMMA) reaction with mixed multipolarity L2 and L2S.

3: Gamma distribution from level J3 following (A,B-GAMMA-GAMMA) reaction with both gammas having pure multipolarity (i.e. no mixing) and first gamma (from J2) is unobserved.

4: Same as Ø1=3 except both gamma transitions are of mixed multipolarity (L2 + L2S and L3 + L3S).

Ø1=21,31,41: Cases 2, 3, 4 respectively are to be immediately followed by case 1 (particle distribution).

111 B distribution as above except that it is calculated for <u>all</u> values of spin and parity of J2 from J0+3+|SP1-SP2| to J0-3+|SP1-SP2|.

220 Gamma distribution as in 01=2 but calculated for all combinations of spin and parity of J2 from J3-2 to J3+2.

221 Gamma distribution of type 220 followed by particle distributions of type 111.

0 STOP the calculation (actually it is preferable to let a "/*" card stop it as then a traceback is always given).

<u>Card 2: TITLE(I), I=1,20 18A4</u>

An alphameric TITLE which should include the energy EL of level J2.

<u>Card 3: J0,J3,EIN,L1MAX,SP1,J4,L2,L2S,DEQ,D2 9F5.2,F10.4</u>

DEQ is the angle interval used for calculation of the cross-sections.

D2: Set at a very large number as 1000.

If one is only interested in <u>one</u> gamma transition, set J4=99.9 and card 4 is <u>not</u> read.

<u>Card 4: (Read only if J4≠99.9) L3,L3S,D3 3F10.4</u>

Again D3 is set at 1000 or larger.

Card 5: J2,L2MAX,SP2,MASS1,MASS2,MASS3,MASS4,Q1,EL   2F5.2,7F10.4

Card 6: Ø2 I1

Option 2 determines how much of the calculations are to be printed out.

Ø2=0 End of input; tabulates products of Racah and Wigner coefficients as given in Sheldon's paper.

=1 (The usual case) Full computation but suppresses printout of the Racah coefficients.

=2 Full computation and full printout.

The following cards contain the transmission coefficients for the incoming and outgoing channels. These are obtainable (e.g.) from N. Davison's Hauser-Feshbach programme.

incoming	Card 7	TO(-) ----- TL1MAX(-)	8F10.6
	8	TO(+) ----- TL1MAX(+)	8F10.6
outgoing	9	TO(-) ----- TL2MAX(-)	8F10.6
	10	TO(+) ----- TL2MAX(+)	8F10.6

Card 11 Ø3   I3

Ø3 is the number of extra exit channels desired in the computation of absolute cross-sections. It will have some effect on the angular distributions and a few (~5) extra channels should be considered, if applicable. The following cards must be repeated Ø3 times if this option is used.

Card 12 Same as card 5 except the data refer to the level populated by the additional channel.

Card 13
Card 14 Same as cards 9 and 10 except they are the transmission coefficients for the extra channel.

NOTE: L2, L2S and L3, L3S must be physically correct, i.e.

$$|J3-J2| \leq L2,L2S \leq J2+J3, \quad |J4-J3| \leq L3,L3S \leq J4+J3$$

A decay to a state of spin zero is always of pure multipolarity.

Description of the Output (assuming Ø2=1) and Remarks

The first one or two pages give back the input data, including the spins, transmission coefficients and extra exit channels in a transparent format, together with the C.O.M. energy of the outgoing particle in each channel.

This is immediately followed by a tabulation of the unnormalized Ferentz-Rosenzweig coefficients (proportional to $F_k(LL'J_3J_2)$) for the relevant gamma ray decay.

Several blank pages may then follow.

The next bit of relevant information given are the calculated population parameters of the various magnetic substates. It is up to the user to determine if these are physically reasonable for his case; this should be done before proceeding. Nonsense parameters should NEVER occur if the program is being properly used.

This is immediately followed by the calculated distribution coefficients (a_2 and a_4) as a function of δ from 0° to ~90° and -0° to ~-90° in 5° steps. The actual mixing ratio (tan δ) is given. NOTE: For a quadrupole transition (in which $|J3-J2| = 2$) agreement with experiment which yields non-zero values of δ are probably not physically reasonable as it implies octupole admixture.

Finally, a multipole ellipse is given with a_2 as abscissa (x-axis) and a_4 as ordinate (y-axis).

<u>Remark</u> The calculated values of the population parameters vary with different transmission coefficients used in the input. As the validity of MANDY depends on the two assumptions

(1) accurate transmission coefficients
(2) statistical model is a good approximation,

it is useful to try a χ^2 fit to the experimental data with different population parameters. This may be done by gridding between the P(m)'s for threshold excitation and those for 200 keV above threshold, using the GRID SEARCH program described elsewhere.

Alternatively, one may simply calculate the χ^2 fit for each set of a_2's and a_4's given by MANDY to the experimental data and apply the usual 0.1% confidence limit test.

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APPENDIX II

The following appendix is a preprint of a paper submitted for publication to Nuclear Physics. The figure and table numbers in the preprint correspond to the following figures and tables in the thesis:

	Appendix	Text
Figure	1	4
	2a,b	4a,b
	3a,b	4d,e
	4	5
Table	1	3

INVESTIGATION OF HIGH SPIN STATES IN ^{55}Fe FROM THE $^{55}Mn(p,n\gamma)$ REACTION†

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INVESTIGATION OF HIGH SPIN STATES IN ^{55}Fe FROM THE ^{55}Mn(p,nγ) REACTION[†]

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Abstract: Electromagnetic decay properties of the low-lying states in ^{55}Fe were studied using the ^{55}Mn(p,nγ)^{55}Fe reaction. The measurement of gamma ray angular distributions together with predictions based on the compound nuclear statistical model has permitted unique spin assignments to be made to the following excited states in ^{55}Fe: 1.316 MeV (7/2), 1.408 MeV (7/2), 2.144 MeV (5/2), 2.211 MeV (9/2), 2.301 MeV (9/2) and 2.470 MeV (3/2). Some multipole mixing ratios were also determined. Comparison is made with the shell-model calculations of Ohnuma and Vervier.

E

NUCLEAR REACTIONS ^{55}Mn(p,nγ), E = 2.75 to 4.25 MeV; measured $\sigma(E_\gamma)$, $\sigma(E_\gamma,\theta)$. ^{55}Fe deduced levels, J, γ-branching, γ-mixing. Natural target.

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1. Introduction

The relatively simple configurations involved in the odd-A N = 29 nuclei, e.g. ^{55}Fe, should enable shell-model calculations with reasonably-sized matrices to reproduce the properties of the low-lying negative parity states of these nuclei. Two such calculations by Vervier [1] and Ohnuma [2] have been carried out. However, the lack of experimental information on many of the low-lying states of these nuclei has hindered a quantitative comparison between experiment and theory. In the present work, we present spectroscopic information on some of the levels of ^{55}Fe below 3 MeV.

Stripping and pickup reactions on the neighbouring iron isotopes [3-5] have yielded ℓ_n transfers to many excited states in ^{55}Fe. For some states, the charged particle angular distributions were "isotropic" from which the authors have concluded large values of ℓ_n and hence high spin. The principal decay modes of the levels populated in the ^{55}Co β^+-decay have been determined by Bauer and Deutsch [6] and Fischbeck et al. [7]. A previous $^{55}Mn(p,n\gamma)$ experiment [8] was limited to a study of levels below 1 MeV.

We have studied the $^{55}Mn(p,n\gamma)$ reaction which excites most of the levels of ^{55}Fe. The negative Q-value of this reaction (-1.014 MeV) allows successive levels in ^{55}Fe to be populated near threshold by suitably varying the bombarding energy. Angular distributions of the gamma decay of each level were then measured; the comparison of these distributions with the predictions of the compound nuclear statistical

model [9,10] then yielded values of the level spins and multipole mixing ratios of the transitions. A similar technique has been used in several recent studies of medium-A nuclei [11-14].

2. Experimental method and analysis

2.1 EXPERIMENTAL METHOD

Targets were prepared by mixing a small amount of powdered manganese into a glue formed by dissolving polyurethane in benzene. The highly viscous mixture was applied to a piece of tantalum metal 0.025 cm thick. This target was placed vertically at 60° to the beam direction and kept in place with a thin aluminum plate which also served as charge collector.

Protons were accelerated by the University of Alberta van de Graaff generator and the beam collimated to 0.15 cm diameter on target. Bombarding energies were selected which corresponded to the reaction proceeding near threshold for each level of interest. The gamma rays were detected with a 45 cc GeLi detector placed at 15 cm from the target; the resolution of the system was typically 3.5 keV FWHM for the 1.33 MeV ^{60}Co full energy peak. Gamma rays were assigned to the decay of a specific level in ^{55}Fe by their appearance in the spectra as the bombarding energy was increased above threshold for that level. The branching ratios given in fig. 1 were obtained by measuring the gamma ray intensities at 60°; the efficiency curve of the GeLi detector was obtained from a study of the gamma rays accompanying the decay of

^{56}Co. Angular distributions were determined from spectra taken at 0, 30, 45, 60 and 90 degrees and normalized to the strong 412 keV γ-ray from the decay of the first excited state ($J=1/2^-$) of ^{55}Fe. Small additional corrections for absorption in the target backing and holder were also made. The angular distributions thus obtained were fitted to a Legendre expansion $W(\theta) = a_0(1+a_2P_2(\cos\theta)+a_4P_4(\cos\theta))$. The Legendre coefficients for each transition are given in Table 1.

2.2 ANALYSIS

The angular distributions were compared with the results of the compound nuclear statistical model of Sheldon [9,10]. In this model, s-wave neutron emission should predominate near threshold leading to an excited state alignment with the substates $|m_j|>J_0$ (with J_0 being the target spin) having small populations. The gamma decay of residual states with $J > J_0$ is predicted to be anisotropic, the extent of the anisotropy depending on the spin J and mixing ratio δ. The predicted distributions were calculated using a computer code MANDY written by Sheldon. The phase convention for the mixing ratio in this code is different from that defined by Rose and Brink [15] but the results presented here have been altered to be consistent with the latter definition.

The proton and neutron transmission coefficients T_ℓ required by MANDY were obtained from a Hauser-Feshbach calculation [16] as well as the code ABACUS II [17] using the Perey [18] optical model parameters. The

two sets of results were only in fair agreement but it was found in the MANDY calculations that a 30% change in the relevant ratio $T_{\ell=1}/T_{\ell=0}$ altered the predicted value of a_2 by less than 3%.

A quantitative measure of goodness of fit between the experimental and predicted angular distributions was obtained using a χ^2 analysis. The 0.1% confidence limit was used to exclude unacceptable fits, as shown in figs. 2 and 3.

3. Results

3.1 THE LEVELS BELOW 1.0 MeV

Previous results[6,7] indicate that the levels at 0, 0.412 and 0.930 MeV have spins $3/2^-$, $1/2^-$, and $5/2^-$ respectively. No confirmation of these assignments could be obtained from the present experiment because of the isotropic γ-ray distributions. These did, however, rule out $J = 7/2$ for the spin of the 0.930 MeV state.

3.2 THE 1.316 AND 1.408 MeV LEVELS

The 1.316 MeV level decays predominantly to the ground state, while the level at 1.408 MeV decays almost equally to the ground and 0.930 MeV state. Fischbeck [7] has also reported an additional weak branch to the 1.316 MeV state.

The ground state transitions of both levels are anisotropic ($a_2 \sim 0.12$). Figure 2 shows the fitted angular distributions and χ^2

plots. On the basis of the 0.1% confidence limit, the only allowed spin assignments are J = 7/2 for both levels.

These assignments are consistent with the (d,p) work of Fulmer [3)] and the (p,d) studies of Glashausser and Rickey [5)] which have led to ℓ_n = 3 assignments for both levels. The state at 1.408 MeV moreover is observed very strongly in the (p,d) reaction, suggesting it is primarily a pickup state.

3.3 LEVELS BETWEEN 1.5 AND 2.2 MeV

The 1.917 and 2.051 MeV states decay only to the ground and first excited states, indicating low spin. This is confirmed by the isotropic distributions, consistent with J = 1/2, 3/2 for both levels, in agreement with the (d,p) work [3,4)] which assigned ℓ_n = 1 angular momentum transfer to both states.

The observed decay of the 2.144 MeV level (20% to both the ground and 1.316 MeV states and 60% to the 0.930 MeV level) is in disagreement with the results of Fischbeck [7)] who only observed the ground state transition. However, they point out that these new transitions occurred in regions of their spectrum where limits on their intensities were larger than the intensity of the ground state transition. Only the 2144 keV gamma ray was observed to be anisotropic ($a_2 = -0.11 \pm 0.03$), consistent with both the J = 3/2 and 5/2 assignments; this together with the known ℓ_n = 3 angular momentum transfer for this level indicates $J = 5/2^-$.

3.4 THE 2.211 AND 2.301 MeV LEVELS

The primary decay of the 2.211 MeV state is via an 803 kev gamma ray to the 1.408 MeV level ($J = 7/2$), with a possible weak branch to the 1.316 MeV level (< 20%). The 2.301 MeV level decays primarily to the 0.930 MeV level ($J = 5/2$) with weak transitions to the 1.316 and possibly 1.408 MeV levels. The decay suggests high spin assignments for both levels.

Fig. 3 shows that both major decays are indeed highly anisotropic ($2.211 \rightarrow 1.408$: $a_2 = -0.54 \pm 0.05$; $2.301 \rightarrow 0.930$: $a_2 = 0.39 \pm 0.05$) and the compound nuclear predictions indicate that both states have $J = 9/2$. The 803-keV gamma ray is of mixed multipolarity with either $\delta = 2.75^{+0.75}_{-0.50}$ or $\delta = 0.19^{+0.07}_{-0.09}$. The 2.301 - 0.930 (9/2 - 5/2) transition is shown to be pure quadrupole.

3.5 LEVELS ABOVE 2.301 MeV

The four levels at 2.470, 2.578, 2.877 and 2.950 MeV all decay predominantly to the ground state; no other branches were observed.

The anisotropic 2470 keV gamma ray ($a_2 = -0.05 \pm 0.02$) together with the reported [3,4] ℓ_n-value of 1 suggests that $J = 3/2$ is the correct assignment for the 2.470 MeV level; the χ^2 fit indeed showed that $J = 1/2$ may be rejected at the 0.1% confidence limit.

The transition from the 2.578 MeV state is nearly isotropic ($a_2 = -0.05 \pm 0.04$) which in conjunction with the measured ℓ_n value of 3 [4] determines the spin of the state to be 5/2; $a_2 = 0.12$ is required

for a 7/2 assignment.

The 2.877 and 2.950 MeV levels were weakly populated in the $^{55}Mn(p,n\gamma)$ reaction. Because of the weak yield and low detector efficiency, statistics for the ground state transitions were poor; no definite spin assignments (besides $J < 7/2$) could be made. Stripping data [3,4] is quite meagre for these levels as they are again weakly populated but apparent ℓ_n-values of 3 are suggested for both levels, limiting their spins to 5/2 or 7/2.

4. Discussion

The theoretical calculations of Vervier [1] and Ohnuma [2] were based on the shell model using a spin-dependent effective p-n interaction. Vervier chose a δ-function for the radial dependence of the p-n force while Ohnuma generalized this to a Gaussian form. The two investigations differed again in the size of the energy matrices considered; Ohnuma including levels of spin up to 13/2 in the odd-A nuclei, Vervier truncating at $J = 7/2$.

The level schemes in fig. 4 show the excellence of the agreement between experiment and Ohnuma's calculations. Noteworthy is his prediction of a triplet near 2 MeV of spins 1/2, 3/2 and 5/2 and of a state of spin 9/2 near 2.3 MeV. These features were all observed in the present work, although we observed two 9/2 states. We note that both calculations predict only one $J = 7/2$ state near 1.5 MeV. However, the state at 1.408

MeV, as shown in Section 3.2 is a pickup state and thus very likely the $f_{\frac{7}{2}}^{-1}$ hole state; it is thus not expected to be reproduced by a calculation taking into account only the effects of particles. Similarly, the 9/2 state at 2.211 MeV which decays strongly to the 1.408 MeV hole state has possibly considerable hole admixture in its configuration. Hence the other 9/2 state at 2.301 MeV is likely the one predicted by Ohnuma, as it decays to the 0.930 (J = 5/2) and 1.316 (J = 7/2) MeV levels, which _are_ included in his theoretical level scheme. However, it is interesting to note that the primary decay is by quadrupole radiation to the J = 5/2 state, rather than by the possibly dipole transition to the J = 7/2 state.

It is a pleasure to thank T.P.G. Carola, E. Wong, W.H. Chung and Dr. B.D. Sowerby for their generous assistance with several aspects of this work, and Dr. E. Sheldon for supplying a copy of the MANDY program to us. One of us (A.A.P.) acknowledges financial assistance from the National Research Council of Canada in the form of a Postgraduate Scholarship.

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